Due Diligence*

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Abstract

Due diligence is common practice prior to the execution of corporate transactions. We propose a model of the due diligence process and analyze its effect on prices, the division of surplus, and efficiency. In our model, if the seller accepts an offer, the winning buyer (the acquirer) has the right to gather information and chooses when (if ever) to execute the transaction. Our main result is that the acquirer engages in "too much" due diligence relative to the social optimum. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting with no due diligence. The optimal contract involves both a price contingent on execution and a non-contingent transfer, resembling features such as earnest money or break-up fees that are commonly observed in transactions involving due diligence.

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1 Introduction

Due diligence is pervasive. It is perhaps most prominent within the context of corporate acquisitions and real estate transactions. However, a period of "discovery" prior to the transfer of ownership and during which the acquirer can gather information extends well beyond these two realms.[1] Practitioners argue that due diligence is critical to ensure a successful transaction.[2] Third-parties (e.g., a financier or board) often require some due diligence prior to deal completion. But is it economically important? What are the welfare implications? More specifically, how does the acquirer's ability to conduct due diligence prior to executing a deal affect the initial terms, the likelihood of deal completion, the total surplus and how it is divided? In this paper, we propose and analyze a model of the due diligence process to answer these questions.

In our baseline setting, we model due diligence as a process that takes place after the seller and winning bidder (henceforth, the *acquirer*) have agreed to the terms of a transaction.[3] There is symmetric uncertainty ex-ante about bidders' (common) value for the seller's asset (and thus whether there are gains from trade). During due diligence, the acquirer uncovers information about this value. Prior to due diligence, buyers make competing price offers for the asset. If the seller accepts one of the offers then the acquirer has the right to conduct due diligence and decide when (if ever) to execute the transaction at the bid price. Thus, the acquirer faces a real option problem during due diligence where the strike price is endogenously determined by the winning bid.

The solution to the acquirers real option problem is to execute the transaction when the probability she assigns to a high asset value is above a threshold. The execution threshold is increasing in the bid: the higher is the bid price, the more due diligence she conducts prior to execution. And a higher execution threshold implies a longer due diligence period and a higher probability of deal failure. As a result, the seller's payoff is not monotonically

[1]To give just one additional example, virtually all retail transactions endow the purchaser with the right to return the item purchased for some period of time. Naturally, the purchaser is likely to learn information about her value for the good during the return period, which influences her decision of whether to exercise the return option.

[2]See e.g., Snow (2011), Lajoux (2010), and Forbes article dated March 27, 2019: "A comprehensive guide to due diligence issues in mergers and acquisitions" (date accessed: August 24, 2020).

[3]See Section 1.1 for justification of this assumption and a discussion of how due diligences is conducted in practice.

increasing in the price. Rather it is hump shaped, thereby illustrating the trade-off between a higher payoff conditional on execution versus a longer and less promising due diligence process. In equilibrium, buyers offer the seller's preferred (interior) price and make a positive expected profit despite being perfectly competitive.

We characterize the unique equilibrium of the model. In equilibrium, due diligence takes place if and only if the (common) prior belief about the value of the asset is below a threshold. Above the threshold, the transaction is executed immediately at the highest price such that the acquirer is willing to forgo performing due diligence. Below the threshold, the equilibrium price (and therefore the execution threshold) is *independent* of the prior belief (Figure 4(a)).

We then compare the equilibrium outcome to two benchmarks: no due diligence and socially optimal due diligence. Because the equilibrium price is above the seller's reservation value, the acquirer's execution threshold is above the social optimal one. As a consequence, there is "too much" due diligence in equilibrium (and too many deals fail) compared to the social optimum. Relative to the no due diligence benchmark, due diligence increases surplus when the prior is low, because without it the transaction would necessarily fail, but decreases social surplus for intermediate priors (i.e., near the socially efficient threshold) because the acquirer is too diligent.

The baseline model makes two simplifying assumptions. First, we restrict the space of contracts to a price contingent on execution. Second, we assume there is symmetric information between buyers and the seller about the assets value. In Sections 3 and 4, we relax both of these assumptions. With symmetric information, the profit maximizing and socially optimal outcome can simultaneously be achieved by enriching the contract space. More specifically, this outcome can be implemented with an up-front (non-contingent) transfer and a contingent price equal to the seller's reservation value. Setting the contingent transfer equal to the seller's reservation value causes the acquirer to fully internalize the cost of delaying execution, while the up-front transfer allows the seller to extract all the surplus.

When the seller is privately informed, acceptance (or rejection) of an offer can signal information to the acquirer, which in turn can influence the acquirer's strategy during due diligence. We first demonstrate that any separating equilibrium involves no trade. The intuition is that if the type is perfectly revealed by the offer accepted, then there is nothing for the acquirer to learn from due diligence. Hence, the acquirer should execute immediately or never. However, if the acquirer executes immediately and a high type is willing to accept the offer then a low type will strictly prefer to accept it.

Using standard refinements, we argue that the high-type optimal equilibrium is focal. This equilibrium involves full pooling and has properties similar to the baseline model. In particular, the contingent price is above the socially optimal level thereby inducing the acquirer to conduct more than the socially optimal level of due diligence. This result is despite the fact that the seller captures all of the surplus from the transaction. The intuition is that an inefficiently high contingent price allows a high-type seller to profitably extract some of the information rents from a low type despite sacrificing total surplus. Unlike in the symmetric information model, the price depends on the prior. As the prior increases, the price decreases and the up-front fee increases.

We consider several extensions of the model. First, we extend the model to allow for (dynamic) bidding during the due diligence process. We provide a sufficient condition under which the timing of due diligence (i.e., whether it takes place before or after an offer is accepted) is irrelevant. This emphasizes that our crucial assumption is that the buyer has the option to conduct due diligence after an offer is accepted, not that this option is necessarily exercised. We then extend the model to allow for common knowledge of gains from trade. In this case, our sufficient condition is violated and the seller may prefer that due diligence be performed before accepting an offer.

Next, we consider the case in which due diligence requires the acquirer to incur a flow cost. A two threshold equilibrium emerges in which the buyer terminates (or cancels) the transaction at a lower threshold after uncovering sufficiently negative information about the seller's asset. With price only offers, the execution threshold remains inefficiently high as is the termination threshold. Allowing for a break-up fee can improve the outcome. In the absence of discounting, break-up fees are equivalent to up-front transfers and either can be used to simultaneously implement the revenue maximizing and socially optimal outcome. Finally, we consider an extension in which the signal-to-noise ratio of the information acquired during due diligence is endogenously determined by the acquirer's effort. Consistent with anecdotal evidence, we find that effort is highest (and learning is fastest) as the deal nears completion.

The remainder of the paper is structured as follows. Section 1.1 discusses when due diligence takes place in practice and Section 1.2 the related literature. Section 2 presents the baseline model and its results. Section 3 shows how up-front transfers can help achieve the socially efficient outcome. Section 4 studies the due diligence process when there is asymmetric information. Section 5 solves several extensions of the baseline model. Section 6 concludes.

1.1 Due Diligence In Practice

Our key assumption is that the acquirer has the right to conduct due diligence after agreeing to terms with the seller. Though clearly a simplification of reality, there are good reasons for this structure, which we discuss in this section. In addition, we discuss the due diligence process of corporate and real estate transactions. We will conclude that indeed a large portion of due diligence takes place after the price has been negotiated.

First, due diligence is costly for the acquiring firm. According to Lajoux (2010): "Some buyers spend millions of dollars identifying every possible risk before signing on the dotted line." Without the terms of a deal in place, potential buyers may be unwilling to invest resources needed to complete the due diligence process in a timely manner.

Second, providing access to proprietary information imposes costs on the target firm. A strategic bidder may use information gathered during due diligence to become more competitive with the target in the case the deal fails (see e.g., Marquardt and Zur (2015) and Wangerin (2019)). A recent example of this behavior occurred in 2019 when Urban Outfitters walked away from a deal to acquire Le Tote, an apparel subscription service, after months of due diligence during which Urban Outfitter's executives visited Le Tote's warehouses multiple times to gain an understanding of its business model. Subsequently, Urban Outfitters launched its own apparel subscription service prompting Le Tote to file a lawsuit against Urban Outfitters alleging a breach of the non-disclosure agreement signed during due diligence.[4]

[4]See Financial Times article dated June 2, 2020: "Is a M&A NDA really just a shadow non-compete?" (date accessed: August 28, 2020)

Due diligence is common practice in Mergers and Acquisitions (M&A). Hansen (2001) and Boone and Mulherin (2007) discuss the takeover process in more detail and Lajoux (2010), Marquardt and Zur (2015), and Wangerin (2019) describe the due diligence that is performed during those deals. Figure 1 illustrates this process. It is common for a buyer to conduct some due diligence prior to signing a confidentiality agreement to see whether she is interested in acquiring the target company. After signing the confidentiality agreement, the buyer performs more due diligence using data supplied by the firm. This phase of a deal is known as the pre-public phase.

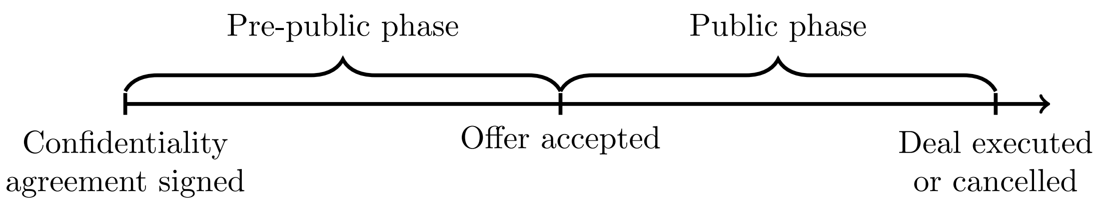


Figure 1: Different phases of an M&A of a company.

After this pre-public phase, the acquirer and target agree upon a price either via an auction or negotiation. The agreement between the acquirer and target often contains warranties and major adverse events clauses, where major events are for example the loss of a large customer or litigation. During the public phase, the acquirer performs additional due diligence by verifying financial information, legal information, existing obligations to suppliers and customers, the state of physical assets etc. While it is unlikeley that all issues are ever fully resolved, the goal of due diligence, according to Snow (2011), is for the acquirer to be "comfortable enough" to go through with the deal and close it. A significant fraction of deals are cancelled during the public phase: Heath and Mitchell (2020) show that between 1986-2018 one in eight deals that is publicly announced is ultimately cancelled.

Failure to conduct proper due diligence can have severe financial consequences for the acquirer. A recent example is the Wirecard scandal. In 2019, Wirecard issued $1bn worth of convertible debt to SoftBank, which it then sold on to investors. A year later it went bankrupt rendering the debt worthless. An undetected accounting fraud, a non-existing cash account, had left a €1.9bn hole in Wirecard's balance sheet.[5] Another example is Hewlett-

[5]See Financial Times article dated June 22, 2020: "SoftBank executives set to lose profits from Wirecard

Packards acquisition of Autonomy in 2011. One year after the acquisition, Hewlett-Packard wrote down $8.8bn in the value of the $11bn acquisition after accounting irregularities that predated the acquisition were discovered. Hewlett-Packard said that no red flags were raised when due diligence was carried out by Deloitte and KPMG.[6]

Due diligence is also common practice in private equity and real estate transactions. In private equity, due diligence involves a significant amount of information production since the target has not been subject to the same public scrutiny or disclosure requirements. In private residential transactions, the buyer typically retains the option to terminate the contract pending the review of seller disclosures, surveys, inspections, and obtaining financing, all of which are forms of due diligence. Commercial real estate deals almost always involve an extended due diligence phase that can last for months. The typical due diligence checklist for commercial real estate transactions includes title and zoning verification, tenant and lease matters, existing and potential legal claims, insurance claims, and physical property inspection. Finally, in transactions involving debt financing, lenders require an independent appraisal. Thus, the need to conduct due diligence can arise from various stakeholders in a transaction.

1.2 Related Literature

Our work relates to the literature on auctions of real options, see Board (2007) and Cong (2018, 2019). One of our contributions is to argue that due diligence expands the scope of this literature. That is, if the acquirer has the right to conduct due diligence prior to execution, then the asset "acquired" when an offer is accepted is in fact a real option. This interpretation leads to results on the efficiency of transactions that include a due diligence clause. Because due diligence is (at least) partly to overcome the adverse selection problem (see the discussion in the previous subsection), the information and payoff structure of our model also fundamentally differs from the existing literature. In particular, we analyze a common-value signaling model with a privately informed seller, whereas this literature considers settings with a private value component in which buyers are privately informed.

trade" (date accessed: August 24, 2020).

[6]See Financial Times article dated November 20, 2012: "HP takes $8.8bn hit over Autonomy" (date accessed: August 24, 2020)

DeMarzo et al. (2005) and Gorbenko and Malenko (2011) analyze auctions where the payments to the seller are contingent on the winning bidder's private information, whereas in our model the transfer depends on the acquirer's execution decision. There is also a large literature that studies information acquisition in auctions prior to bidding (Matthews, 1984; Stegeman, 1996; Persico, 2000; Shi, 2012), whereas in our model information is acquired after bidding.

We also contribute to the theoretical literature on M&A.[7] Within this space, our paper is most closely related to the M&A literature that studies the takeover (auction) mechanism, see Bulow and Klemperer (1996), Hansen (2001), Ye (2007), Quint and Hendricks (2018), and Gorbenko and Malenko (2018, 2019). Our contribution is in studying the impact of the due diligence during the public phase, which is especially relevant in takeover auctions.[8]

There is a key timing difference in this paper compared to earlier work by Daley and Green (2012, 2020) where learning takes place before the buyer and seller have agreed to terms and ends as soon as the seller accepts an offer. In this paper, the buyer has the option to continue to learn by performing due diligence after terms of the deal have been set. The difference in timing leads to fundamentally different predictions. Perhaps most notable among them is that inefficient delays can occur even with symmetric information and common knowledge of gains from trade (see Section 5.2). Whereas trade is both fully efficient and immediate when learning ends once terms are reached.

2 Baseline Model

In this section, we propose and analyze a simple model of due diligence, which includes the essential ingredients and highlights most of the intuition for our results.

There is one seller and (at least) two competitive buyers. The seller owns an asset, which is either of high or low value. We denote the asset type by $\theta \in \Theta = \{L, H\}$. The seller has

[7] See Betton et al. (2008) and Eckbo et al. (2019) for excellent reviews of this literature.

[8] Ye (2007) and Quint and Hendricks (2018) study the impact of the due diligence during the pre-public phase, which takes places before a binding is offer is made, while we focus on the role of the due diligence during the public phase, which takes place after a binding offer is made. Marquardt and Zur (2015) and Wangerin (2019) mention that when a company is sold via an auction due diligence during the public phase is especially important since the target does not want to disclose its private information to all bidders.

a reservation value of k for the asset regardless of its type. Buyers have a common value for the asset, which we denote by V_θ. We assume there are gains from trade if and only if the asset type is high: $V_H > k \geq V_L$ and normalize $V_L = 0$ without loss of generality. Asset type is unknown to both buyers and the seller. All agents are risk-neutral and discount their cash flows at the rate r.

The model takes place in continuous time. At $t = 0$, the seller holds an auction for the asset. Each buyer makes a price offer and the seller selects the winning buyer. After the auction, the winning buyer can perform due diligence and decides when (if ever) to execute the transaction and complete the deal. During due diligence, the buyer gathers information about the type of the asset. If the winning bid is P and the buyer executes the transaction at date τ, then the seller's (net) payoff is $e^{-r\tau}(P - k)$ and the buyer's payoff is $e^{-r\tau}(V_\theta - P)$. If the transaction is never executed then all players' payoffs are zero. Figure 2 illustrates a sample timeline of the game.

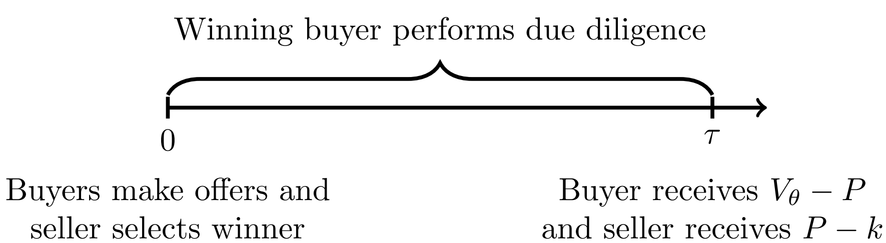


Figure 2: Sample Timeline of the Game.

2.1 Learning

The seller and buyers have a common prior $q_0 = \mathbb{P}_0[\theta = H]$. During due diligence, the winning buyer observes information about the asset's type from a Brownian information process

$$dX_t = 1_{\{\theta=H\}}dt + \frac{1}{\phi}d\hat{B}_t, \tag{1}$$

where $\hat{B}_t$ is a standard Brownian motion on the canonical probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and ϕ is the signal-to-noise ratio or *quality* of the information process. A higher ϕ makes the information process less noisy and therefore more informative about θ. The information process $\{X_t\}_{t\geq 0}$ generates a filtration $(\mathcal{H}_t)_{t\geq 0}$. Bayes' rule then implies that the beliefs that the asset's type is high evolve according to

$$dq_t = \phi q_t(1-q_t)\left(dX_t - q_t dt\right) = \phi q_t(1-q_t)dB_t,$$

where B_t is a Brownian motion on the probability space $(\Omega \times \Theta, \mathcal{F} \times 2^{\Theta}, \mathcal{P} \times \upsilon)$ where υ is the measure over Θ defined implicitly by q_0.

2.2 Strategies and Equilibrium Concept

The game can be divided into two stages. In the first stage, buyers simultaneously make offers to the seller and the seller decides which offer to accept. In the second stage, the winning buyer—henceforth, the *acquirer*—conducts due diligence until she deems it optimal to execute the transaction. The second stage is a proper subgame, which we refer to as the due-diligence subgame.

Suppose the winning bid price is P. In the due-diligence subgame, the acquirer chooses a stopping time to maximize her expected discounted payoff

$$F_B(q|P) = \sup_{\tau} \mathbb{E}_q\left[e^{-r\tau}(V_\theta - P)\right].$$

The acquirer's strategy in the due-diligence subgame is therefore a collection of stopping times, which are indexed by the winning bid price and denoted by $\tau(P)$.

The seller must take into account the acquirer's strategy in the due-diligence subgame when deciding what offer to accept. Let $F_S(q|P)$ denote the seller's expected payoff from accepting an offer of P

$$F_S(q|P) = \mathbb{E}_q\left[e^{-r\tau(P)}(P-k)\right].$$

Therefore, the seller chooses offer i if $F_S(q|P_i) \geq \max\{0, F_S(q|P_j)\}$. Clearly, the seller will reject any offer below k and the buyer never completes the transaction if the price is larger than V_H. Therefore, we can restrict attention to bids in the interval $[k, V_H]$. Our equilibrium concept is subgame perfect Nash equilibrium, henceforth referred to simply as equilibrium.[9]

2.3 Equilibrium Construction

We solve for the equilibrium by backward induction. Given any price P, the winning buyer faces a stopping problem of when to complete the transaction. The solution to this problem is to complete the transaction as soon as the belief exceeds a threshold.

Lemma 1 (Acquirer-Optimal Execution). *Given any price $P \in (0, V_H)$, the acquirer completes the deals as soon as q_t exceeds*

$$b(P) = \frac{1}{1 + \frac{1-\underline{q}(P)}{\underline{q}(P)} \times \frac{u-1}{u}} = \frac{\underline{q}(P)u}{u - 1 + \underline{q}(P)} > \underline{q}(P) \tag{2}$$

where $u = \frac{1}{2}(1 + \sqrt{1 + 8r/\phi^2})$ and $\underline{q}(P) = \frac{P-V_L}{V_H-V_L}$. The acquirer conducts due diligence when beliefs are below $b(P)$. That is,

$$\tau^*(P) = \inf \{t > 0 | q_t \geq b(P)\}.$$

The optimal acquisition threshold depends on the product of two terms. The first term is the belief such that the expected value of the asset is equal to the price $\underline{q}(P)$, which is increasing in P. This belief is akin to the strike price on a call option: for beliefs above $\underline{q}(P)$ the option is in the money. The second term depends only on $\gamma \equiv \phi^2/r$, which captures the informativeness (or speed) of learning per unit cost of time. The higher is γ, the greater is the option value from due diligence and the higher is the execution threshold. As $\gamma \to 0$, the value of conducting due diligence goes to zero and $b(P) \to \underline{q}(P)$. Figure 3 illustrates the acquirer's value function, the optimal execution threshold, and its decomposition.

Having characterized the acquirer's strategy in the due diligence subgame, let us now consider the first stage when the price is determined. Because buyers are identical and

[9] The role of subgame perfection is that it requires the acquirer to play optimally in the due-diligence subgame for any possible winning bid and not just the bid that is accepted on the equilibrium path.

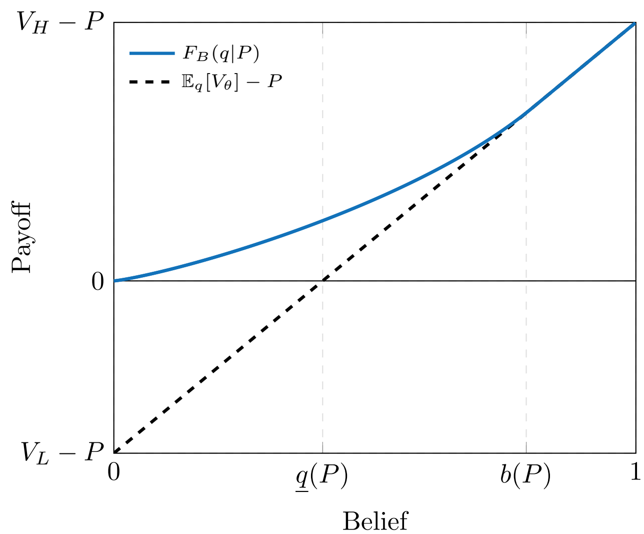


Figure 3: Solution to the Acquirer's Problem in the Due Diligence Subgame.

perfectly competitive, the equilibrium price maximizes the seller's expected payoff. Clearly, a higher price is good for the seller conditional on execution. However, because a higher price implies a higher execution threshold, it also implies more delay and a higher chance of deal failure. When the price is low (i.e., close to the seller's reservation value), the marginal cost of a higher price is relatively small and outweighed by the higher payoff conditional on execution. For higher prices, the marginal cost of a higher price outweighs the marginal benefit. As a result, the seller's payoff is initially increasing in P and eventually decreasing in P.

To characterize the seller-optimal price, let $P_0(q)$ denote the highest price at which the acquirer is willing to forego due diligence and execute the transaction immediately given the belief is q. In order to induce immediate execution, the seller must provide some rents to the acquirer. Thus, $P_0(q) < E_q[V_\theta]$. A closed-form expression for P_0 can be obtained by inverting $b(P)$ from equation (2) to get

$$P_0(q) = \frac{(1-q)uV_L + q(u-1)V_H}{u-q}.$$

Next, consider the hypothetical (stopping) problem in which the seller chooses when to accept $P_0(q_t)$:

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau}(P_0(q_\tau) - k) \right]. \tag{SP-hypothetical}$$

The value under the solution to this problem provides an upper bound on the seller's equilibrium payoffs. The solution necessarily involves an upper belief threshold, $q^* \in \mathbb{R}$, above which it is optimal to stop. The following assumption says that it is never optimal to stop below q^*.

Assumption 1. *There exists a $q^* \in (0, 1)$ such that the solution to (SP-hypothetical) is $\tau = \inf\{t \geq 0 : q_t \geq q^*\}$.*

Assumption 1 is not directly about primitives of the model. The next lemma provides sufficient conditions on primitives for it to hold.

Lemma 2. *Fixing all other parameters, Assumption 1 is satisfied if either:*

(i) $k \geq \bar{k}$ for some $\bar{k} \in (V_L, V_H)$.

(ii) $\gamma \geq \bar{\gamma}$ for some $\bar{\gamma} > 0$.

Assumption 1 can fail if k is small enough and γ is neither too large, nor too small. Our numerical analysis suggests the region of the parameter space in which Assumption 1 can fail is rather small: $\bar{k}$ is less than 5 percent of V_H across all γ.[10] However, it necessarily fails if there is common knowledge of gains from trade (i.e., $k < V_L$). We analyze this case in Section 5.2, but assume that Assumption 1 holds until then.

Lemma 3 (Seller-Optimal Price)**.** *Let $P^* \equiv P_0(q^*)$. The seller optimal price, denoted $P_S(q)$, is given by*

$$P_S(q) = \begin{cases} P^* & \text{if } q \leq b(P^*) \\ P_0(q) & \text{if } q > b(P^*) \end{cases}$$

[10]Our proof of Lemma 2 establishes that $\bar{k}$ depends only on γ and V_H.

For $q \geq b(P^*)$, the seller prefers an offer that induces the acquirer to forego due diligence and execute the transaction immediately. For $q < b(P^*)$, the seller prefers the acquirer to conduct due diligence with the hope of selling for P^* in the future rather than settle for $P_0(q)$ today. Naturally, competition among buyers drives the winning offer to the seller's preferred price.

Proposition 1. *There exists a unique equilibrium. In it,*

(i) The winning offer is the seller-optimal price, $P_S(q_0)$.

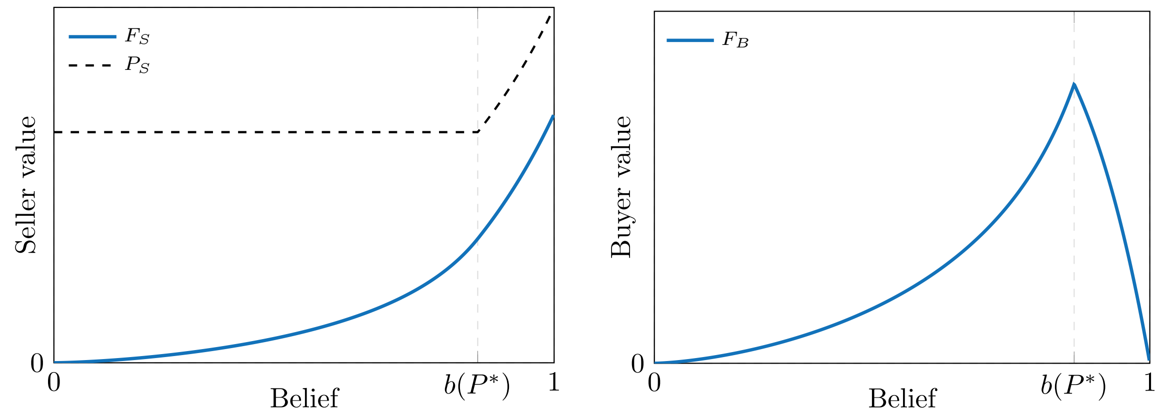
(ii) In the due-diligence subgame, the acquirer plays according to $\tau^*(P)$.

(iii) There is a period of due diligence if and only if $q_0 < b(P^*)$.

Figure 4 illustrates the equilibrium payoffs and price as well as the likelihood of deal completion as it depends on the initial belief. Notice that the seller-optimal price is constant for all $q < b(P^*)$, corresponding to the region in which due diligence takes place. Because of this structure, the seller's equilibrium payoff is the same as in (SP-hypothetical). More importantly, this structure further implies that whether due diligence takes place before or after (or during) bidding is payoff irrelevant. We formally demonstrate this result in Section 5.1, where we extend the model to allow for dynamic bidding. Thus, the key assumption for our results is that *the winning buyer has the option to conduct (more) due diligence after her offer is accepted*. If potential buyers can conduct due diligence prior to or during negotiations then the option to conduct more after an offer is accepted may not be exercised. However, the equilibrium payoffs remain unchanged.

2.4 Implications

How does the acquirer's ability to conduct due diligence affect the equilibrium outcome? Does due diligence improve efficiency or lead to unnecessary delays? To answer these questions, it is instructive to compare the equilibrium outcome to two benchmarks.



(a) Equilibrium Price and Seller Value

(b) Buyer Value

Figure 4: Equilibrium Prices and Payoffs as a Function of the Belief.

2.4.1 No Due Diligence

Consider a benchmark without due diligence: if the seller accepts the buyer's offer then the transaction is immediately executed. In this case, the seller's expected payoff is increasing in the price. As a result, competitive buyers offer a price equal to the expected asset value and the seller accepts the offer when it is above her reservation value.[11] Without the ability to conduct due diligence, the acquirer makes zero profit.

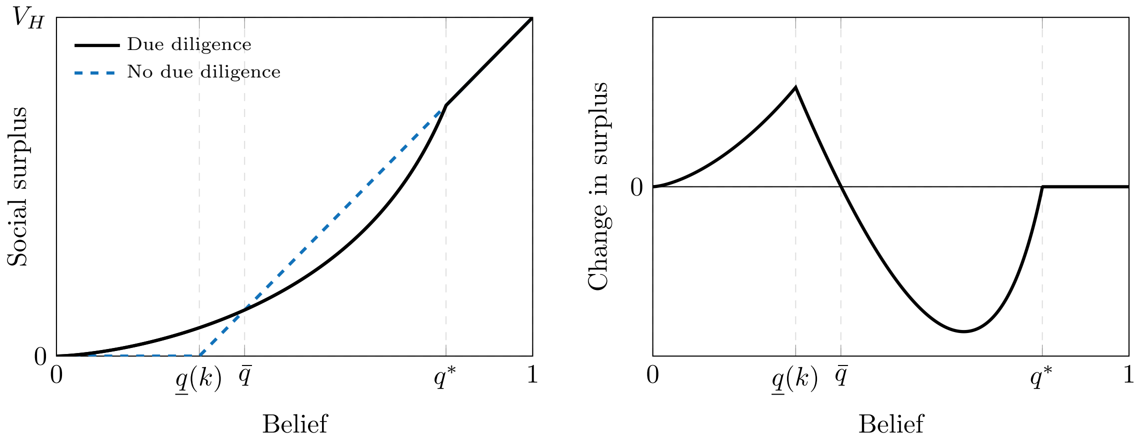
Corollary 1 (Acquirer Profits)**.** *The acquirer's ability to perform due diligence allows him to extract a positive surplus in equilibrium:* $F_B(q_0|P_S(q_0)) > 0$ *for all* $q_0 \in (0, 1)$.

Note that this result holds regardless of whether the acquirer actually conducts any due diligence along the equilibrium path. The mere option to conduct due diligence is what facilitates surplus extraction.

Figure 5(a) plots the social surplus (seller plus acquirer value) both with and without due diligence. Figure 5(b) plots their difference, which is the net surplus that can be attributed to due diligence. These figures suggest that due diligence improves surplus for low beliefs but hinders it for intermediate beliefs. The following proposition formalizes this result.

Proposition 2 (Social Surplus from Due Diligence)**.** *There exists a threshold* $\bar{q} < b(P^*)$ *such that*

[11] This is also the outcome that obtains in the limit as $\phi^2/r \to 0$.



(a) Social Surplus

(b) Difference Social Surplus

Figure 5: Impact of Due Diligence on Surplus.

(i) For $q_0 < \bar{q}$, *due diligence increases social surplus.*

(ii) For $q_0 \in (\bar{q}, b(P^*))$, *due diligence decreases social surplus.*

(iii) For $q_0 \geq b(P^*)$, *due diligence does not change social surplus.*

Intuitively, allowing for due diligence increases surplus when the deal would necessarily fail without it. However, because the acquirer is too diligent relative to the social optimum (Proposition 3), allowing for due diligence when it is (close to) socially optimal to execute immediately decreases surplus.

2.4.2 Social Optimum

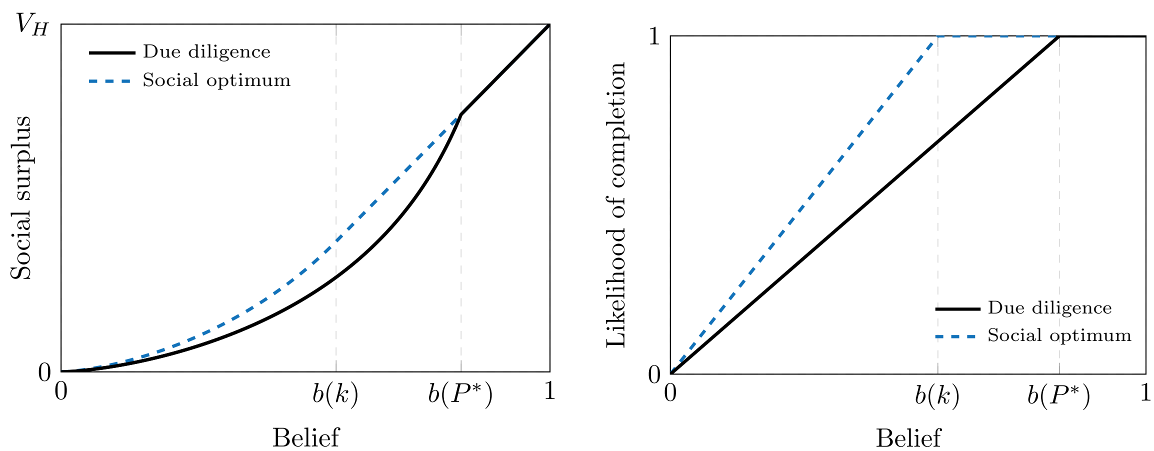
Consider a social planner who does not know the asset's type, but has the ability to learn by observing the information generated during the due diligence. The planner's problem is to choose a stopping time to solve

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau}(V_\theta - k) \right].$$

Notice that the planner's problem is identical to the acquirer's problem except that the strike price of the option is the seller's reservation value rather than the winning bid.

Proposition 3 (Social Optimum)**.** *The socially optimal execution threshold is $b(k)$, which is strictly less than $b(P^*)$. Therefore, in equilibrium, the acquirer conducts "too much" due diligence (and the probability of deal failure is too high) relative to the social optimum.*

Intuitively, this result derives from the fact that the acquirer only captures part of the surplus, and therefore does not internalize the full cost of delay (or deal failure). To induce efficient execution, the price paid conditional on executing the transaction should be set equal to k. But if the price is k, then the seller captures none of the surplus. Competition among buyers drives the price above k, which in turn leads to inefficient execution.



(a) Social Surplus in Equilibrium Compared to the Social Optimum

(b) Likelihood of Deal Completion

Figure 6: Impact of Due Diligence on Surplus.

3 Optimal Mechanisms

One way to overcome the inefficiency identified in Proposition 3 is to enrich the space of contracts to allow for transfers that are not contingent on execution. Doing so allows buyers to transfer surplus to the seller without inefficiently distorting the execution decision. To formalize this idea, suppose that a contract, C, consist of a pair $C \equiv (U, P)$, where $U \geq 0$ is an up-front unconditional transfer and P is the price paid contingent on execution. In this case, we obtain the following result.

Proposition 4. *Consider the baseline model with the enriched contract space consisting of both an up-front transfer and a price contingent on execution. Then, the unique equilibrium involves: the seller accepting an offer of* $(U, P) = (F_B(q_0|k), k)$ *if* $q_0 < b(k)$ *and immediate execution if* $q_0 \geq b(k)$*. In either case, the acquirer's expected payoff is zero and due diligence is socially efficient.*

In effect, the acquirer purchases the option to buy the firm at a price of k and, in doing so, transfers all of the surplus to the seller through the up-front payment. The contingent price is set so that the acquirer correctly internalizes the social cost/benefit of conducting due diligence.

Proposition 4 is related to Board (2007), who studies the sale of a real option in a private value setting. There are two differences. First, Board (2007), assumes that the seller's reservation value evaporates after the option is transferred to the buyer regardless of whether the option is executed, whereas in our setting the seller foregos his reservation value only if and when the option is executed. Therefore, Board (2007) concludes that the socially optimal mechanism does not involve a contingent transfer. Second, our baseline model is simpler in that it involves pure common values. Since the seller can extract all of the surplus, there is no tension between revenue maximization and surplus maximization. Thus, unlike in Board (2007), the seller optimal and socially optimal mechanisms coincide.

Although allowing for up-front transfers resolves the distortion in our baseline model, they are not a panacea. For instance, as we demonstrate in the next section, when the seller is privately informed, contingent transfers serve as an effective screening mechanism. As a result, the equilibrium contract involves contingent transfers that are larger than is socially optimal, which results in "too much" due diligence similar to Proposition 3 (see Proposition 11). Relatedly, Board (2007) shows that when buyers have private information about their value, the revenue maximizing mechanism relies too heavily on contingent transfers thereby inducing the acquirer to inefficiently postpone exercising the option.

4 Asymmetric Information

In this section, we extend our analysis to a model in which the seller is privately informed about the asset type. This setup serves not only as a robustness check, but also to demonstrate that with the adverse selection problem, inefficiency persists even when the contract space allows for non-contingent transfers. Specifically, we now assume that the seller knows the type of its asset $\theta \in \{L, H\}$ while, as before, the asset's type is unknown to buyers. In addition, we allow the seller's reservation value to depend on the asset's type k_θ with $V_H > k_H > k_L > V_L = 0$.

As in Section 3, we pay particular attention to contracts, $C = (U, P)$, that specify U as the (unconditional) up-front transfer and P as the price paid contingent on execution, neither of which depend on the information obtained during due diligence.[12] Let $F_\theta(q|P) = \mathbb{E}_q^\theta\left[e^{-r\tau(P)}(P - k_\theta)\right]$ denote the value to a type-θ seller derived from the due diligence subgame with contracted price P. F_θ differs from F_S in that the expectation about what the acquirer learns during due diligence depends on θ.

4.1 Constrained Efficiency

Let us first revisit the problem of a social planner. As before, the first-best outcome is to execute the transaction immediately if $\theta = H$ and never if $\theta = L$. We refer to the *constrained efficient* outcome as the solution to the problem of a planner who has the same access to information as the acquirer (i.e., the planner does not know θ but can learn about it through due diligence). The planner's problem can be written as

$$\sup_\tau \mathbb{E}_q\left[e^{-r\tau}(V_\theta - k_\theta)\right].$$

As before, it is socially optimal to delay executing the deal until beliefs reach a threshold, denoted b_{SP}, after which it is socially optimal to execute the deal.

Proposition 5. *The constrained efficient execution threshold is* $b_{SP} \equiv \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$. *Moreover, the constrained efficient execution threshold is acquirer-optimal under the con-*

[12] In Section 4.3, we consider the case in which the terms of the transaction can depend on the information obtained during the due diligence process X.

tingent price $P_{SP} \equiv \alpha V_H + (1-\alpha)V_L \in [k_L, k_H]$, *where* $\alpha = \frac{k_L - V_L}{(V_H - V_L)-(k_H - k_L)}$. *That is,* $b(P_{SP}) = b_{SP}$.

While constrained efficient execution in the due diligence subgame is acquirer-optimal with a contingent price of P_{SP}, the difference in seller types creates a hurdle to the implementation of the constrained efficient outcome. Because $P_{SP} < k_H$, the high-type seller earns a loss at the time of execution. To satisfy the (high-type) seller's participation constraint, this loss needs to be overcome with a transfer from the acquirer. However, when the initial belief is pessimistic, even transferring the acquirer's entire expected surplus is insufficient to compensate for the high type's expected loss.

Proposition 6. $F_H(q_0|P_{SP}) + F_B(q_0|P_{SP}) \geq 0$ *if and only if*

$$q_0 \geq \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{V_H - V_L}\right\}. \tag{3}$$

Hence, there exists a contract (U, P) *that achieves the constrained efficient outcome and satisfies both buyer and seller participation constraints if and only if* (3) *is satisfied.*

4.2 Equilibrium

We now analyze the game in which the seller posts a contract $C = (U, P)$. Under this formulation, the model's first stage is a signaling game due to the seller's private information.[13] We therefore modify our solution concept from subgame perfect Nash equilibrium to Perfect Bayesian Equilibrium (PBE).[14] In equilibrium, for any posted contract C, buyers update their belief about θ from q_0 to $\tilde{q}(C)$ that is consistent with the seller's posting strategy. If accepted, the seller's total expected payoff from $C = (U, P)$ is $U + F_\theta(\tilde{q}(C)|P)$.

As is common in signaling games, there exist many PBE, due to the flexibility afforded to off-path beliefs. Consider, for example, the constrained efficient PBE (assuming (3) is satisfied) in which both seller types pool on the contract with $P = P_{SP}$ and attempt to

[13] Our results are unchanged if instead buyers make public contract offers to the seller. While the arguments are slightly different and more complicated in this case, the key component is that the seller's choice of contract to accept can still serve as a signal of his private information.

[14] See (Fudenberg and Tirole, 1991, pp. 331-333).

extract all the surplus, $U = U_{SP}(q_0) \equiv F_B(q_0|P_{SP})$. Since both types post the same contract, buyers do not learn anything from the seller's posting (i.e., $\tilde{q}(U_{SP}(q_0), P_{SP}) = q_0$).

How reasonable is the constrained efficient equilibrium? This question is usually addressed by asking, what *should* buyers believe if an off-path signal is observed? In our case, suppose the seller unexpectedly posts C' consisting of a slightly higher price and a slightly lower up-front. Which type is more willing to make this tradeoff? A higher price induces (weakly) more due diligence. Because the high type expects better news, a higher price is "more likely" to help the high type. What is a "reasonable" belief to hold, then, after observing C'? The relatively weak equilibrium refinement known as *divinity* (Banks and Sobel, 1987) requires only that C' does not lead the buyer to think worse of the seller's type: the belief following C' should be no lower than the prior q_0.[15] But if there is no deterioration of belief from posting C', then it may well be attractive to the high type to do so.

In fact, it can be shown that the constrained efficient PBE satisfies divinity if and only if it maximizes the high-type seller's payoff among all PBE. Partially motivated by this result, we identify the high-type optimal PBE, hereafter the *HTO equilibrium*. Unlike many signaling games, the high type in our model never does best by separating. In any separating equilibrium, the high type posts a contract so unfavorable to the buyer that she never executes *even though* the acquirer is sure the seller has a high type asset (otherwise the low type would prefer to imitate him). Of course, in a separating equilibrium, the low type does not transact as well.[16]

Proposition 7. *In any separating PBE, the transaction is never executed.*

Henceforth, we refer to such equilibria, in which there is no trade, as *trivial*. Proposition 7 implies that the HTO equilibrium must involve at least some pooling. In fact, it involves full pooling.

Proposition 8. *Any HTO equilibrium involves full pooling on a single contract. Moreover, a*

[15] Divinity is the static analog of the *belief monotonicity* criterion used in Swinkels (1999) and Daley and Green (2012).

[16] Moreover, the *intuitive criterion*—the canonical refinement used to generate separation in signaling games—has no refining power in our game. This fact is due to the signal in our model (i.e., posting a contract) having no direct cost. For example, no matter how large a price the seller demands, the worst that can happen to him is for the contract be rejected, resulting in a payoff of zero.

nontrivial PBE is a HTO equilibrium if and only if it satisfies both divinity and the undefeated criterion (Mailath et al., 1993).[17]

Below we provide sufficient conditions for the high-type optimal equilibrium to be unique. Numerical results indicate that uniqueness is generic, but it remains unproven.

4.2.1 Characterizing the High-type Optimal Equilibrium

Because the HTO equilibrium is full pooling, the buyers' belief after the offer is posted remains their prior, q_0, and the contract chosen solves

$$\max_{U,P} \; U + F_H(q_0|P)$$
$$s.t. \quad F_B(q_0|P) - U \geq 0.$$

The constraint ensures that buyers make a non-negative profit and are therefore willing to make the offer. Clearly, the constraint binds at any solution, reducing the problem to

$$\max_{P} F_H(q_0|P) + F_B(q_0|P). \tag{4}$$

If the solution to (4) involves immediate execution (i.e., $b(P) \leq q_0$), then there is no distinction between contingent and up-front transfers and the optimal P is not uniquely defined (only $U + P$ is pinned down). The following assumption guarantees a unique solution for all q_0.

Assumption 2. $\frac{k_H}{V_H} \geq \left(1 - \frac{u}{2}\right)$.

We generalize our results when Assumption 2 does not hold in Proposition 11.

Proposition 9. *Under Assumption 2, the HTO equilibrium is unique and*

(i) features immediate execution if $q_0 \geq b(k_H) > b_{SP}$,

[17]The undefeated criterion is arguably the most well known refinement that is not based on Kohlberg and Mertens (1986)'s notion of *stability* (note that both divinity and the intuitive criterion are based on stability). While neither refinement selects a unique PBE in our model, it is appealing that HTO equilibria are the sole survivors of these refinements with different foundations.

(ii) features a price $P_H(q_0) > k_H$, and the acquirer conducts due diligence, if $q_0 < b(k_H)$.

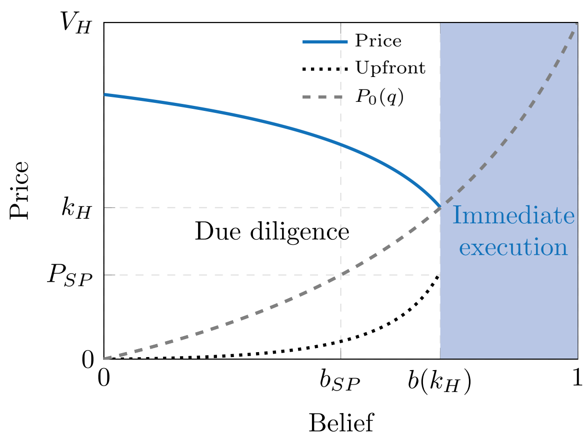
Compared to the constrained efficient outcome, the HTO equilibrium features a higher price and "too much" due diligence/delay. The HTO equilibrium also requires a higher initial belief in order to forgo due diligence and immediately execute the deal. Figure 7(a) illustrates. Intuitively, the greater price benefits the high type, who expects good news and to complete the deal more quickly, at the expense of the low type (Figure 7(b)). Unlike in the baseline model, this inefficiency persists even with the unconditional up-front transfer.

Also in contrast to the baseline model, the high-type optimal price depends on the prior belief. Perhaps surprisingly, this price is decreasing in the belief (Figure 7(a)). The intuition is that the high type faces a tradeoff between generating total surplus (which is maximized at $P = P_{SP}$), and extracting surplus from the low type with a higher price. When buyers believe the low type is more likely, extraction becomes relatively more important.

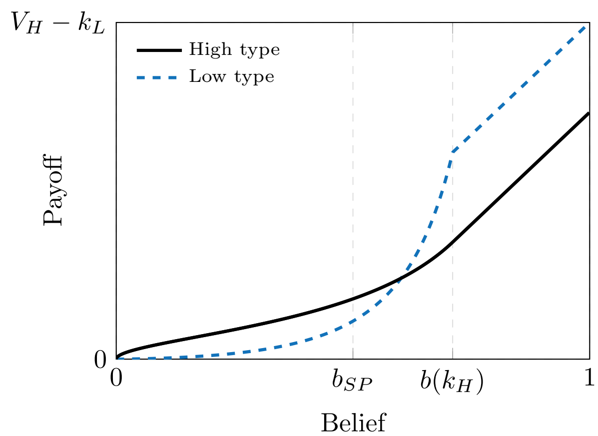
Proposition 10. *Under Assumption 2,*

(i) The price $P_H^*(q_0)$ is continuous and decreasing at all $q_0 < b(k_H)$, and $P_H^* \to k_H$ as $q_0 \to b(k_H)$.

(ii) The upfront $U_H^*(q_0)$ is continuous and increasing at all $q_0 < b(k_H)$, and $U_H^* \to \mathbb{E}[V_\theta | q = b(k_H)] - k_H$ as $q_0 \to b(k_H)$.



(a) Equilibrium Price



(b) Equilibrium Payoff

Figure 7: Equilibrium Price and Payoff when there is Asymmetric Information.

Assumption 2 fails if and only if $k_H < V_H/2$ and γ is sufficiently large, in which case we have the following characterization.

Proposition 11. *If Assumption 2 fails, there exists a pair* $(\overline{q}_H, \underline{q}_H)$, $\overline{q}_H \geq \underline{q}_H > b(k_H) > b_{SP}$, *such that the HTO equilibrium*

(i) is unique and features immediate execution if $q_0 > \overline{q}_H$,

(ii) is unique, features a price $P_H^*(q_0) > k_H$, *and the acquirer conducts due diligence, if* $q_0 < \underline{q}_H$.

Recall that under Assumption 2, $\overline{q}_H = \underline{q}_H = b(k_H)$. So, there are two differences without Assumption 2. First, we only have proven that $\overline{q}_H \geq \underline{q}_H$, though in all of the numerical examples we have analyzed, $\overline{q}_H = \underline{q}_H$. Second, and of more economic interest, with a higher quality information process, the high-type now requires a prior strictly above $b(k_H)$ in order to forgo due diligence. Proposition 10 continues to hold at all $q_0 < \underline{q}_H$, with the exception that the price does not limit to k_H, but instead to a higher price.

4.3 When Due Diligence Information is Contractible

Thus far, we have assumed that the information acquired during due diligence is exogenously revealed through the stochastic process X prior to execution. In reality, the information acquired during due diligence is endogenous, potentially manipulable, and may be privately observed by the acquirer. Therefore, it is unrealistic to assume that parties could contract directly on X, without requiring additional incentive compatibility constraints. Nevertheless, the case with contractible X is an interesting benchmark.

It turns out that with contractible X and unbounded transfers, it is possible to get arbitrarily close to the first-best outcome, meaning trade occurs almost immediately with the high type and never with the low type, while simultaneously giving all of the surplus to the high-type seller.[18] However, to accomplish this approximation the contingent price must be arbitrarily sensitive to a small amount of information. Moreover, we encounter a Mirrleesian existence problem.

[18] In fact, any division of the first-best surplus between the high and low type can be approximated.

It is probably not surprising that allowing the execution price to depend on the entire history facilitates a separating equilibrium in which trade occurs with positive probability. What is more surprising is that we can obtain a separating equilibrium in which trade occurs (almost) *immediately*. There are two instruments that history contingent prices facilitate which are used in the construction. First, a wedge in the execution price depending on whether due diligence reveals positive or negative information. Following the revelation of negative information, the acquirer executes the transaction but at less favorable price for the seller than if due diligence reveals positive information. This wedge makes the contract relatively less attractive to the low type. However, it also gives incentive to the acquirer to delay until enough negative information arrives, which constrains the ability to induce efficient execution. To circumvent this issue, one can use a non-stationary contingent transfer. Once enough good (or bad) information is revealed, the price is fixed thereafter independent of any information that follows.

Formal Setup of the Problem The space of contracts are now pairs $\mathcal{C} = \{U, (P_t)_{t\geq 0}\}$ where U is the up-front transfer and P_t is the execution-contingent price which can depend on the entire history of information, $\mathcal{H}_t = \sigma(\{X_s : 0 \leq s \leq t\})$ (i.e., P_t is measurable with respect to $\mathcal{H}_t$).

Consider an arbitrary contract $\mathcal{C}$ and execution rule τ. Abusing notation, let $F_\theta(\mathcal{C}, \tau) = \mathbb{E}_q^\theta[e^{-r\tau}(P_\tau - k_\theta)]$ denote the type-θ seller's payoff in the due-diligence subgame (i.e., ignoring the up-front transfer) and let $F_B^\theta(\mathcal{C}, \tau) = \mathbb{E}_q^\theta[e^{-r\tau}(V_\theta - P_\tau)]$ denote the acquirer's payoff in the due-diligence subgame conditional on the seller's type being θ.

The high-type optimal separating contract solves

$$\sup_{\mathcal{C},\tau} F_H(\mathcal{C}, \tau) + U$$

$$s.t. \quad F_L(\mathcal{C}, \tau) + U \leq 0 \tag{5}$$

$$F_B^H(\mathcal{C}, \tau) - U \geq 0 \tag{6}$$

$$\tau \in \arg\sup_T \mathbb{E}_q^H[e^{-rT}(V_H - P_T)]. \tag{7}$$

The first constraint, (5), says that the low type prefers to reject the contract.[19] The second constraint, (6), says that the acquirer does not lose money. The third constraint, (7), ensures that τ is consistent with acquirer optimization in the due-diligence subgame.

It is easy to see that both (5) and (6) must bind in any solution. Thus, $F_B^H = U$ and the problem can be written as

$$\sup_{\mathcal{C},\tau} F_H(\mathcal{C},\tau) + F_B^H(\mathcal{C},\tau)$$

$$s.t. \quad F_L(\mathcal{C},\tau) = -F_B^H(\mathcal{C},\tau) \tag{8}$$

$$\tau \in \arg\sup_T \mathbb{E}_q^H[e^{-rT}(V_H - P_T)]. \tag{9}$$

Note that the objective is equal to the total surplus, $F_H(\mathcal{C},\tau)+F_B^H(\mathcal{C},\tau) = E^H[e^{-r\tau}(V_H - k_H)]$. So, the problem boils down to finding the stopping time which maximizes $E^H[e^{-r\tau}]$, subject to finding prices that satisfy the two constraints.

Approximate Solution If the acquirer immediately executed the transaction and given the high type prefers to accept, the low type would also prefer to accept. Thus, to dissuade the acquirer from immediate execution, we set the price above V_H until the first time that q_t exits an ε-neighborhood around the initial starting point. Denote this time by $\tau(\varepsilon) \equiv \inf\{t : |q_t - q_0| \geq \varepsilon\}$.

Proposition 12. *For any $\varepsilon > 0$, there exists a $\mathcal{C}_\varepsilon$ such that $(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ satisfies (8) and (9) in which the high-type seller extracts all of the surplus.*

Thus, Proposition 12 shows that we can get arbitrarily close to the first-best outcome: as $\varepsilon \to 0$, $E^H[e^{-r\tau}(V_H - k_H)] \to V_H - k_H$. However, to obtain the first-best outcome requires $E^H[e^{-r\tau}] = 1$, hence $\mathcal{Q}^H(\tau = 0) = 1$. But $\mathcal{Q}^H$ and $\mathcal{Q}^L$ are equivalent measures (i.e., mutually absolutely continuous with respect to each other) and so $\mathcal{Q}^L(\tau = 0) = 1$. If $\tau = 0$ w.p.1. under both measures then their respective payoffs in the due-diligence subgame must also be the same (up to differences in reservation value) and therefore the low type would strictly prefer to accept the contract, a contradiction.

[19] As we will see, it is unnecessary to provide any surplus to the low-type seller to achieve separation.

5 Other Extensions

In this section, we consider several extensions of the model. First, we look at the implications of dynamic bidding and common knowledge of gains from trade. Second, we allow for costly due diligence, deal termination, and explore the role of break-up fees. Third, we allow for the information quality during due diligence to be endogenously determined by costly effort. For brevity, we consider only the symmetric information model.

5.1 The Timing of Due Diligence

In practice, some portion of due diligence takes place before or during negotiations between buyers and the seller. How does this affect the equilibrium outcome? As we show below, our results are unchanged when we allow for bidding while simultaneously performing due diligence. In other words, it is the *option* to conduct (more) due diligence after her offer is accepted that is important for our results, not the precise timing of when due diligence takes place.

The game again consists of two phases and the second phase is identical to the baseline model. However, the first stage is now dynamic. In the first stage, competitive buyers conduct due diligence while simultaneously making offers. To facilitate comparison with the baseline model, we assume that all buyers observe the same information process in the first stage (as given in (1)). Let ν be the time at which the seller accepts an offer and denote the offer by P_ν. After the seller accepts an offer, the due-diligence subgame ensues, i.e., the winning buyer can continue to perform due diligence and decide when if ever to execute the transaction at the price P_ν. Applied to corporate takeover auctions, one can interpret bids made prior to ν as indicative bids made during the "pre-public" phase of the negotiation (Ye (2007), Quint and Hendricks (2018), and Liu and Officer (2019)). Whereas at date ν, the seller asks potential buyers to submit formal bids. Figure 8 illustrates a sample timeline of the game with dynamic bidding.

The main insight of this section is that, under Assumption 1, allowing for due diligence prior to (or during) bidding does not substantively alter the model's predictions.[20]

[20]This finding is in contrast to Cong (2019), where the seller may optimally choose to (inefficiently) delay

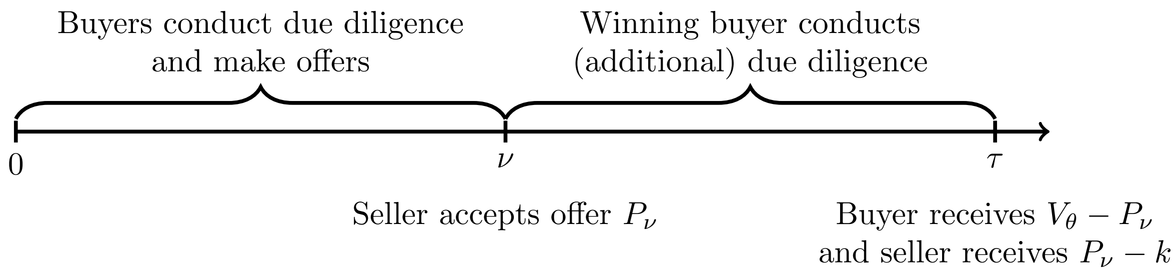


Figure 8: Sample Timeline of the Game with Dynamic Bidding.

Proposition 13 (Dynamic Bidding Irrelevance)**.** *Suppose that Assumption 1 holds. Then, in any equilibrium with dynamic bidding, the payoffs of the seller and the winning buyer are the same as they are in the equilibrium of the model with static bidding.*

For intuition, the seller's equilibrium payoff in the model with static bidding is same as the solution to the hypothetical stopping problem of when to accept $P_0(q_t)$ (see (SP-hypothetical)). With dynamic bidding, the seller can do no worse; by definition buyers are willing to offer $P_0(q_t)$ and execute immediately. The surprising part is that the seller can also do no better. This is because, the seller optimal price is independent of the prior in the due diligence region. Therefore, regardless of what information is uncovered during due diligence, the seller has no desire to renegotiate the price. Hence, she has nothing to gain by waiting to accept an offer.

For $z < b(P^*)$, the price offers and acceptance time are not uniquely determined with dynamic bidding. In one such equilibrium, buyers offer $P_0(q_t)$ at all t and $\nu = \tau(P^*)$, meaning that buyers only conduct due diligence *prior* to reaching an agreement and never *after* the seller accepts. In another equilibrium, buyers offer $P_S(q_t)$ at all t, $\nu = 0$, and due diligence only takes place after the seller accepts. However, in any equilibrium the execution price and execution time are P^* and $\tau(P^*)$ respectively. Hence, both the seller and the acquirer's payoffs are identical.

Assumption 1 is sufficient to ensure dynamic bidding irrelevance. It is also necessary in the sense that if it fails, then there exists priors such that the seller prefers dynamic bidding. However, rather than explore this relatively small region of the parameter space (see the discussion following Lemma 2), we extend the parameter space to allow for common

the date of the auction in order to reduce (privately-informed) bidder's information rents.

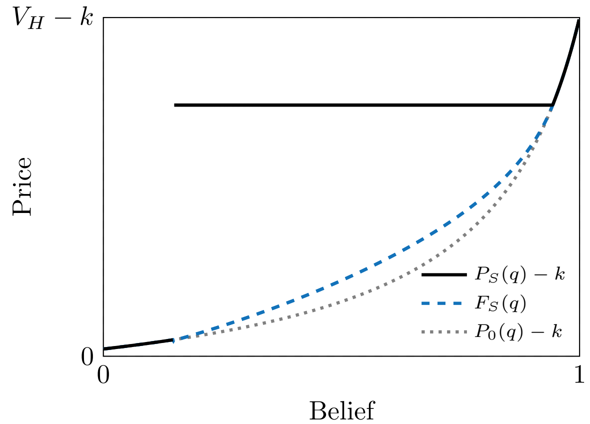


Figure 9: Equilibrium Price and the Seller's Payoff with CKGT

knowledge of gains from trade (i.e., $k < V_L$), which necessarily violates Assumption 1.

5.2 Common Knowledge of Gains from Trade

Thus far, we have assumed that there are no gains from trade when $\theta = L$. In this subsection, we explore the implications of relaxing that assumption. When $k < V_L$, there is common knowledge of gains from trade (CKGT). It is therefore socially optimal to execute the transaction immediately for all prior beliefs. However, we have already seen the equilibrium without CKGT features excessive due diligence. Under what conditions does a similar result obtain with CGKT? The answer is provided by the following proposition.

Proposition 14. *Suppose that $k < V_L < V_H$. Then, there exists a $\underline{\gamma} > 0$ such that the equilibrium with static bidding is as follows.*

(i) If $\gamma > \underline{\gamma}$, there exist two thresholds $q_a < q_b$ such that

(a) For $q_0 \in (q_a, q_b)$, the winning offer is $P_0(q_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(q_b))$.

(b) For $q_0 \notin (q_a, q_b)$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

(ii) If $\gamma \leq \underline{\gamma}$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

When the speed of learning relative to the cost of delay is below $\underline{\gamma}$, due diligence is too costly to warrant appeal from the seller. More interestingly, when γ is above $\underline{\gamma}$, a two-threshold equilibrium emerges.[21] For beliefs above the upper and below the lower threshold, there is no due diligence. But for intermediate beliefs, the seller prefers a price higher than the one that would induce the buyer to execute immediately. Thus, even when there is no social motive for due diligence, the seller can benefit from inducing the acquirer to conduct due diligence as a means of reducing the acquirer's rents.

One novel implication of the equilibrium structure in Proposition 14 is that dynamic bidding is no longer payoff equivalent to static bidding.

Proposition 15 (Dynamic Bidding Relevance with CKGT)**.** *Suppose that $k < V_L$. Then, there exists a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$, the seller strictly prefers dynamic bidding. Moreover, with dynamic bidding, all due diligence takes place prior to the seller accepting an offer.*

To understand why the seller's prefers dynamic bidding, suppose that the initial prior is $q_0 \in (q_a, q_b)$ and thus the winning offer with static bidding is $P_0(q_b)$. If bad news is revealed during due diligence such that the belief drops below q_a, then the seller optimal price also drops (see Figure 9) and the seller would therefore like to renegotiate the price down to induce the acquirer to execute immediately.[22] With static bidding, this is not feasible. However, with dynamic bidding, the seller can accomplish the same outcome by delaying acceptance until due diligence has been effectively completed (i.e., the belief reaches either q_a or q_b) prior to accepting an offer.

Proposition 15 has implications for whether we should expect to see due diligence take place before or after the seller and acquirer agree to terms. In a strategic acquisition, the primary motivation for the buyer to acquire the target is due to synergies, which corresponds to a gain from trade with the target regardless of the value of the firm. Whereas, a financial bidder is typically only interested in acquiring targets that are undervalued. Our model suggests that in strategic acquisitions, there is an advantage to conducting due diligence prior to terms being reached (e.g., in the "pre-public" phase), whereas there is no such advantage

[21]When Assumption 1 fails and $k \geq V_L$, the equilibrium with static bidding is similar to Proposition 14, part (i), with the addition of a third belief threshold $q_c \leq q_a$ such that for $q_0 < q_c$, the offer is $P_0(q_c)$ and the acquirer conducts due diligence until $\tau^*(q_c)$.

[22]Of course, by allowing the acquirer to renegotiate, the seller would subject herself to a hold-up problem.

for purely financial transactions. Of course, providing multiple potential buyers with access to sensitive information about the firm may impose costs on the seller (non-disclosure agreements notwithstanding). Thus, our results suggest that more of the due diligence will take place in the "pre-public" phase for deals with strategic buyers compared to financial buyers. Another prediction relates to the likelihood of deal completion. With dynamic bidding and CKGT, the deal is completed with probability one. A testable implication is that deals with financial buyers should be more likely to fail.

5.3 Costly Due Diligence and Break-up Fees

Until now we assumed that due diligence does not consume any of the acquirer's resources. In practice, there are costs to performing due diligence. For example, the acquirer typically needs to hire an accountant to verify financial statements and/or inspectors to evaluate the condition of physical capital. When due diligence is costly, the acquirer may prefer to terminate the transaction if enough negative information is revealed rather than continue to conduct due diligence. In this section, we extend the baseline model to allow for costly due diligence and deal termination.

In particular, assume that the acquirer pays a flow cost c until it completes the deal, which happens at date τ, or terminates the deal, which happens at date ζ. The acquirer sets the completion and termination time to maximize her payoff

$$F_B(q|P) = \sup_{\tau,\zeta} \mathbb{E}_q \left[\int_0^{\min\{\tau,\zeta\}} e^{-rt} c dt + \mathbb{I}_{\{\tau<\zeta\}} e^{-r\tau} (V_\theta - P) \right].$$

Fixing any $P \in (k, V_H)$, the acquirer's optimal strategy is to complete the deal for beliefs above an upper threshold $\hat{b}(P)$, to terminate the deal for beliefs below a lower threshold $\hat{a}(P)$, and to conduct due diligence for beliefs in between.[23] Our main results from the baseline model can be extended to the case with costly due diligence. For any $P > k$, the acquirer's execution threshold remains above the socially efficient execution threshold while the lower termination threshold is above the socially efficient threshold. Intuitively,

[23]The upper and lower thresholds can be determined by solving a system of four boundary conditions (value matching and smooth pasting at each boundary) and four unknowns (two constants in the acquirer's value function and the two boundaries).

because the acquirer does not capture all of the surplus, she "gives up" prior to when it socially optimal to do so. In that sense, there is both inefficient execution and inefficient termination.

Interestingly, the seller benefits from a marginal increase in due diligence costs starting from $c = 0$. Furthermore, the profit maximizing and socially efficient outcome can be obtained with an upfront transfer and contingent price $P = k$ (i.e., Proposition 4 still holds). If the acquirer is unable to offer an upfront transfer (e.g., due to financing/liquidity constraints), then a break-up fee (paid from the acquirer to the seller conditional on termination) can improve the seller's payoff by inducing both earlier execution and later termination. In fact, absent discounting (i.e., if $r = 0$) break-up fees are equivalent to up-front transfers and the optimal mechanism can be implemented with a price/break-up fee pair. More precisely, if $r = 0$ then a contract $C = (U, P)$ is formally equivalent to a contract with a break-up fee of $B = U$ and a contingent price $P + U$.

Corollary 2. *Consider the symmetric information model with costly due diligence and no discounting (i.e., $r = 0$). The socially optimal and seller profit maximizing mechanism can be implemented with a break-up fee $B = F_B(q|k)$ and a contingent price $P = k + B$.*

5.4 Endogenous Effort

Thus, far, we have assumed that the quality of information obtained during due diligence is exogenously fixed. In practice, the acquirer can choose how much effort to invest in gathering information during the due diligence process, which determines the quality of the information she collects per unit of time and therefore impacts the time until deal completion. We show below that the closer the acquirer is too completing the deal the more effort she exerts.

We assume that each instant the acquirer can set the (squared) quality ϕ^2 of the due diligence process and that higher quality due diligence requires more effort. These effort cost come in the form of a quadratic flow cost

$$c(\phi^2) = \frac{1}{2}\phi^2.$$

The acquirer sets her due diligence quality ϕ_t^2 and deal completion time τ to maximise her payoff

$$F_B(q|P) = \sup_{\tau,\{\phi_t^2\}_{t\geq 0}} \mathbb{E}_q\left[\int_0^\tau e^{-rt}c(\phi_t^2)dt + e^{-r\tau}(V_\theta - P)\right].$$

Proposition 16. *The optimal due diligence quality is*

$$\phi^2(q) = \sqrt{2rF_B(q|P)}.$$

The due diligence quality, and therefore due diligence effort, is increasing in beliefs since the acquirer's expected discounted payoff is increasing in beliefs.

6 Conclusion

Due diligence is common practice prior to the execution of corporate transactions. We propose a model of the due diligence process and analyze its effect on prices, the division of surplus, and efficiency. If the acquirer has the right to conduct due diligence (and terminate the contract if the result of due diligence is unsatisfactory) then the asset she acquires conditional on the seller's acceptance is a real option, which has important economic implications for trading outcomes.

Most notably, we show that the acquirer engages in "too much" due diligence relative to the social optimum and can extract a positive surplus even with perfect competition among potential buyers. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting with no due diligence. The optimal mechanism involves both a price contingent on execution as well as a non-contingent transfer, resembling features such as earnest money or termination fees that are commonly observed in transactions involving due diligence.

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A Appendix

In the appendix, we work with beliefs or the log-likelihood of beliefs

$$z = \log\left(\frac{q}{1-q}\right).$$

The dynamics of the log-likelihood of beliefs are given by

$$dZ_t = \frac{\phi^2}{2}\left(2\frac{e^{Z_t}}{1+e^{Z_t}} - 1\right)dt + \phi dB_t.$$

A.1 Baseline Model

Proof of Lemma 1. We construct the value function $F_B(z|P)$ given the threshold strategy $b(P)$ (with a slight abuse of notation using the q-space threshold in z-space) and show that it solves the Hamilton-Jacobi-Bellman equation

$$0 = \max\left\{-rF_B(z|P) + \frac{\phi^2}{2}\left(2\frac{e^z}{1+e^z} - 1\right)F_B'(z|P) + \frac{\phi^2}{2}F_B''(z|P), (\mathbb{E}_z[V_\theta] - P) - F_B(z|P)\right\}.$$

For $z \geq b(P)$, we have that

$$F_B(z|P) = \mathbb{E}_z[V_\theta] - P$$

by construction. Furthermore, we have that

$$-rF_B(z|P) + \frac{\phi^2}{2}\left(2\frac{e^z}{1+e^z} - 1\right)F_B'(z|P) + \frac{\phi^2}{2}F_B''(z|P) = \frac{r(P + e^z(P - V_H) - V_L)}{1+e^z}.$$

This term has the same sign as

$$P + e^z(P - V_H) - V_L,$$

which is decreasing in z since $P < V_H$ and at $b(P)$ is equal to

$$P + e^{b(P)}(P - V_H) - V_L = \frac{-P + V_L}{u-1} < 0.$$

This result implies that $F_B(z|P)$ solves the Hamilton-Jacobi-Bellman equation for $z \geq b(P)$.

For $z < b(P)$, the value function $F_B(z|P)$ solves

$$0 = -rF_B(z|P) + \frac{\phi^2}{2}\left(2\frac{e^z}{1+e^z} - 1\right)F_B'(z|P) + \phi^2 F_B''(z|P)$$

by construction and therefore is given by

$$C_1 \frac{e^{uz}}{1+e^z} + C_2 \frac{e^{\hat{u}z}}{1+e^z}$$

with

$$u = \frac{1+\sqrt{1+\frac{8r}{\phi^2}}}{2} > 1$$

and $\hat{u} < 0$. Furthermore, it satisfies the boundary conditions

$$\begin{aligned} F_B(b(P)|P) &= \mathbb{E}_z[V_\theta] - P, \\ \lim_{z\to-\infty} F_B(z|P) &= 0. \end{aligned}$$

This implies that

$$\begin{aligned} C_1 &= e^{-b(P)u}\left(-\left(1+e^{b(P)}\right)P + e^{b(P)}V_H + V_L\right), \\ C_2 &= 0. \end{aligned}$$

The final step is showing that $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$. Define

$$G(z) = \frac{\partial\left(F_B(z|P) - \left(\mathbb{E}_z[V_\theta] - P\right)\right)}{\partial z}.$$

We know that $G(b(P)) = 0$ by construction. Furthermore, $G(z)$ has the same sign as

$$\frac{(1+e^z)^2}{e^z} G(z)$$

and we know that

$$\frac{\left(1+e^{b(P)}\right)^2}{e^{b(P)}} G(b(P)) = 0.$$

Furthermore,

$$\frac{\partial\left(\frac{(1+e^z)^2}{e^z}G(z)\right)}{\partial z} = u\left(e^z+1\right)(P-V_L)e^{(u-1)z}\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^{-u} > 0,$$

which shows that $\frac{(1+e^z)^2}{e^z}G(z) < 0$ for $z < b(P)$ and therefore that $G(z) < 0$ for $z < b(P)$. This result implies that

$$F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$$

for $z < b(P)$, and therefore the Hamilton-Jacobi-Bellman equation is also satisfied for $z < b(P)$.

The value function we constructed based on the threshold $b(P)$ solves the Hamilton-Jacobi-Bellman equation for any z and therefore it is the solution to our optimal stopping problem. □

Assume from now onwards for the baseline model that $V_L = 0$. Define the operator of the stopping problem

$$\sup_{\tau} \mathbb{E}_q \left[e^{-rt}(P_0(q) - k) \right]$$

as

$$\mathcal{A}(q,t) = e^{-rt} r \left(-(P_0(q) - k) + \frac{\gamma}{2} q(1-q) P_0''(q) \right).$$

Lemma A.1. *There exists a $\bar{k}$ such that for $k \geq \bar{k}$ the operator $\mathcal{A}(q,t)$ satisfies single crossing with respect to q.*

Proof. Observe that

$$\frac{\gamma}{2} q(1-q) P_0''(q) = -\frac{2(q-1)^2 q^2 V_H}{(q-u)^3} \geq 0$$

and at $q = 1$ this term is zero. Furthermore, $P_0(q)$ is strictly increasing in q. This implies that

$$-(P_0(q) - k) + \frac{\gamma}{2} q(1-q) P_0''(q)$$

attains its minimum at $q = 1$.

The operator is increasing in k. Therefore, there exists a $\bar{k}$ such that for $k \geq \bar{k}$ the operator satisfies single crossing where $\bar{k}$ is the lowest local minimum of

$$-P_0(q) + \frac{\gamma}{2} q(1-q) P_0''(q)$$

for $q \in (0,1)$, if it exists. If no local minimum exists then $\bar{k} = 0$. □

Lemma A.2. *Assume $\frac{k}{V_H} > \frac{1}{4}$. Then there exists a $\bar{\gamma}$ such that for $\gamma \geq \bar{\gamma}$ the operator $\mathcal{A}(q,t)$ satisfies single crossing with respect to q.*

Proof. The constant $u > 1$ is decreasing in γ since

$$u = \frac{1 + \sqrt{\frac{8}{\gamma} + 1}}{2} > 1.$$

The operator $\mathcal{A}(q,t)$ has the same sign as

$$\hat{\mathcal{A}}(q) = -(q-u)^3 \frac{e^{rt}}{r} \mathcal{A}(q,t)$$

since $u > 1$ for any $\gamma \in [0, \infty)$. We know that

$$\hat{\mathcal{A}}(0) = u^3 k > 0,$$
$$\hat{\mathcal{A}}(1) = (u-1)^3(k - V_H) < 0.$$

We have

$$\hat{\mathcal{A}}'(0) = -3ku^2 - (-1+u)u^2 V_H < 0.$$

Furthermore,

$$\hat{\mathcal{A}}''(q) = 6ku + 24q^2 V_H + 4(1 + (-1+u)u)V_H + q(-6k - 6(3+u)V_H)$$

is a quadratic equation with for u sufficiently close to one

$$\hat{\mathcal{A}}''(0) = 6ku + 4(1 + (-1+u)u)V_H > 0,$$
$$\hat{\mathcal{A}}''(1) = 6k(-1+u) + 2(5 + u(-5+2u))V_H > 0.$$

This result implies that $\hat{\mathcal{A}}''(q)$ changes sign either never or twice for $q \in [0, 1]$.

For u sufficiently close to one, using the fact that $\frac{k}{V_H} > \frac{1}{4}$, we get that

$$\hat{\mathcal{A}}'''(q) = -6(k + (3 - 8q + u)V_H) < 0.$$

As a result $\hat{\mathcal{A}}''(q) > 0$, which implies that $\hat{\mathcal{A}}'(q)$ crosses zero at most once (it is first negative and then positive or always negative) and as a result $\hat{\mathcal{A}}(q)$ crosses zero once from above.

The previous steps imply that for u sufficiently close to one $\mathcal{A}(q, t)$ satisfies single crossing with respect to q. □

Lemma A.3. *Assume $\frac{k}{V_H} \leq \frac{1}{4}$. Then there exists a $\bar{\gamma}$ such that for $\gamma \geq \bar{\gamma}$ the operator $\mathcal{A}(q, t)$ satisfies single crossing with respect to q.*

Proof. For u sufficiently close to one

$$\hat{\mathcal{A}}'(0) = -3ku^2 - (-1+u)u^2 V_H < 0,$$
$$\hat{\mathcal{A}}'(1) = -(-1+u)^2(3k + (-3+u)V_H) > 0.$$

Furthermore, $\hat{\mathcal{A}}'(q)$ is a cubic equation in q, which crosses zero at most three times. These results imply that $\hat{\mathcal{A}}'(q)$ crosses zero either once or three times for $q \in [0, 1]$.

For u sufficiently close to one

$$\hat{\mathcal{A}}''(0) = 6ku + 4(1 + (-1+u)u)V_H > 0,$$
$$\hat{\mathcal{A}}''(1) = 6k(-1+u) + 2(5 + u(-5+2u))V_H > 0.$$

For $u = 1$, the largest solution to $\hat{\mathcal{A}}''(q) = 0$ is given by

$$\frac{3k + 12V_H + \sqrt{9k^2 - 72kV_H + 48V_H^2}}{24V_H} \leq \frac{3 + 12 + \sqrt{48}}{24} < 1.$$

This implies that for u sufficiently close to one there exists a $\hat{q} << 1$ such that for $q > \hat{q}$ $\hat{\mathcal{A}}''(q) > 0$. Therefore, for $q > \hat{q}$ we have that $\hat{\mathcal{A}}'(q)$ is either always positive or initially negative and then positive. This implies that for $q > \hat{q}$ $\hat{\mathcal{A}}(q)$ crosses zero at most once, which therefore also holds true for $\mathcal{A}(q,t)$.

For $q \in [0, \hat{q})$ and for u sufficiently close to one the operator is positive because for $u = 1$ the operator becomes

$$k + \frac{q^2 V_H}{1-q} > 0.$$

The last two steps imply that for u sufficiently close to one a $\mathcal{A}(q,t)$ satisfies single crossing. □

Proof of Lemma 2. Lemma A.1, A.2, and A.3 imply that $\mathcal{A}(q,t)$ satisfies single crossing and therefore the optimal stopping strategy for the accompanying stopping problem is a threshold strategy, see (Dixit et al., 1994, Ch. 5). □

Proof of Lemma 3. We know that

$$\sup_{\tau} \mathbb{E}_q \left[e^{-rt}(P_0(q) - k) \right] \geq \max_{p \in [P_0(q), V_H]} F_S(q|P)$$

and using the equilibrium price causes these two to be the same and therefore the equilibrium price is optimal. □

Proof of Proposition 1. From Lemma 1 it follows that the threshold stopping rule $b(P)$ is optimal for any price P. Given this stopping rule, Lemma 3 shows that a seller payoff is maximised when buyers offer the unique price

$$P_S(z) = \begin{cases} P^* & \text{if } q \leq b(P^*) \\ P_0(q) & \text{if } q > b(P^*) \end{cases}.$$

Given that there are at least two competitive buyers, they offer the unique price that maximizes the seller's value. This shows that there is a unique equilibrium. □

A.2 Social Surplus

Proof of Proposition 2. Define $\underline{z} < b(0)$ as the solution to

$$\mathbb{E}_{\underline{z}} \left[V_\theta - k \right] = 0.$$

We know that for $z \leq \underline{z}$

$$F_S(z|P_S(z)) + F_B(z|P_S(z)) > 0 = \max \left\{ \mathbb{E}_z \left[V_\theta - k \right], 0 \right\}.$$

For $z \in (\underline{z}, z^*)$, we can differentiate the difference in social surplus

$$\frac{\partial \left(\mathbb{E}_z \left[V_\theta - k \right] - F_S(z|P^*) - F_B(z|P^*) \right)}{\partial z}.$$

This derivative has the same sign as

$$\frac{(1+e^z)^2}{e^z}\frac{\partial\left(\mathbb{E}_z\left[V_\theta-k\right]-F_S(z|P^*)-F_B(z|P^*)\right)}{\partial z}$$
$$=\frac{e^{(u-1)z}\left((u-1)e^z+u\right)\left(\frac{P^*u}{(u-1)(V_H-P^*)}\right)^{-u}\left(P^*uV_H-k(P^*+(u-1)V_H)\right)}{(u-1)(P^*-V_H)}+V_H.$$

This function crosses zero at most once and therefore it implies that the sign of

$$\frac{\partial\left(\mathbb{E}_z\left[V_\theta-k\right]-F_S(z|P^*)-F_B(z|P^*)\right)}{\partial z}$$

flips at most once on the interval $(\underline{z}, z^*)$.

At z^*

$$F_S(z^*|P^*)-F_B(z^*|P^*)=\mathbb{E}_{z^*}\left[V_\theta-k\right].$$

Furthermore, in some left neighbourhood of $z^*(>b(0))$ in the case without due diligence the deal is completed, which maximizes social surplus, while in the case with due diligence the deal is delayed until beliefs reach z^*, which is socially suboptimal. Therefore, in some left neighborhood of z^*

$$F_S(z|P^*)+F_B(z|P^*)<\mathbb{E}_z\left[V_\theta-k\right].$$

The last three steps imply that at $\underline{z}$ it is socially optimal to perform due diligence, in some left neighborhood of z^* it is suboptimal to perform due diligence, and on $(\underline{z}, z^*)$ the derivative of the difference in the social surplus switches sign at most one. This directly implies that there exists a $\bar{z} \in (\underline{z}, z^*)$ such that for $z \in (\underline{z}, \bar{z})$ doing due diligence increases the social surplus while for $z \in (\bar{z}, z^*)$ it decreases the social surplus.

For $z \geq z^*$, in both cases the deal is completed directly and the difference in social surplus is zero. □

Proof of Proposition 3. The proof of the first statement is analogous to the proof of Lemma 1 after replacing P with k. That $k < P^*$ follow from the fact that if $P = k$ then the seller's payoff is zero whereas for any $P > k$, the seller's payoff is strictly positive. Hence, $P^* > k$. □

Proof of Proposition 4. Clearly, the candidate is an equilibrium. Since due diligence is efficient and the seller extracts all of the surplus, any other offer will either (i) be less attractive to the seller and therefore rejected, or (ii) earn negative expected payoff for the acquirer. In either case, a profitable deviation for potential buyers does not exist. For uniqueness, note that with upfront payments, there is now transferable utility and thus by the standard argument for Bertrand competition, buyers must earn zero profit (otherwise they can increase U by ϵ and win with probability one). Second, note that for any other candidate equilibrium, there exists an offer that the seller prefers to accept and earns a strictly positive expected profit. Thus, a profitable deviation exists. □

A.3 Proofs for Model with Asymmetric Information (Section 4)

We first prove Propositions 5, 6, and 12. We then establish preliminaries for the signaling model of Section 4.2, and prove all the results therein.

Proof of Proposition 5. The proof is analogous to the proof of Lemma 1 after replacing P with k_θ. □

Proof of Proposition 6. The proof has several steps. First, we need to show that there exists a $\hat{q}$ such that $F_H(q|P_{SP}) + F_B(q|P_{SP}) > 0$ if and only if $q > \hat{q}$. For $q = 1$, we know that direct execution happens and the payoff is $V_H - k_H > 0$. For low enough beliefs, the buyer always delays since there are only gains from trade with the high type and $P_{SP} > V_L$. From $F_H(q|P_{SP}) + F_B(q|P_{SP})$ assuming there is delay it follows that there exists a $\tilde{q} > 0$ such that for $q \in (0, \tilde{q})$ this value is negative.

Observe that $F_H(q|P_{SP}) + F_B(q|P_{SP})$ assuming there is delay crosses zero only once. The same holds true for $F_H(q|P_{SP}) + F_B(q|P_{SP})$ assuming there is direct execution. Furthermore, $F_H(q|P_{SP}) + F_B(q|P_{SP})$ is continuous in q. This shows that there exists a $\hat{q} \in (0, 1)$ such that $F_H(q|P_{SP}) + F_B(q|P_{SP}) > 0$ if and only if $q > \hat{q}$.

There are two possible cases for $\hat{q}$ now:

1. At $F_H(\hat{q}|P_{SP}) + F_B(\hat{q}|P_{SP}) = 0$ there is immediate execution. In this case $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$. At $\frac{k_H - V_L}{V_H - V_L}$ immediate execution is optimal if and only if

$$\frac{k_H - V_L}{V_H - V_L} \geq b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$$

which is the case if and only if

$$u \geq \frac{k_H - V_L}{k_H - k_L}$$

since b_{SP} is decreasing in u (less informative due diligence means sooner execution).

2. At $F_H(\hat{q}|P_{SP}) + F_B(\hat{q}|P_{SP}) = 0$ there is delay. In this case $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$. At $\frac{u(k_H - k_L)}{V_H - V_L}$ delay is optimal if and only if

$$\frac{u(k_H - k_L)}{V_H - V_L} \leq b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}. \tag{10}$$

Observe that

$$k_H(u-1) - u(k_L + V_H - V_L) + V_H = (k_H - V_H)(u-1) - u(k_L - V_L) < 0.$$

Therefore, (10) holds if and only if

$$\frac{u(k_H - k_L)}{V_H - V_L}(k_H(-1+u) + V_H - u(k_L + V_H - V_L)) - u(V_L - k_L) \geq 0.$$

which is a quadratic equation in u with as solution $u = 0$ and $u = \frac{k_H - V_L}{k_H - k_L}$. Furthermore, the terms infront of u^2 are negative which implies that for $u \in \left(0, \frac{k_H - V_L}{k_H - k_L}\right) \frac{u(k_H - k_L)}{V_H - V_L} \leq b_{SP}$. As a result for $u \leq \frac{k_H - V_L}{k_H - k_L}$ delay is optimal.

To conclude, we know that if $u \leq \frac{k_H - V_L}{k_H - k_L}$ holds then delay is optimal and $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$ while if it fails then $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$. This is equivalent to saying $\hat{q} = \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{k_H - k_L}\right\}$. $\square$

Proof of Proposition 12. We prove this result by construction. Define P_t^e as follows:

$$P_t^e = \begin{cases} \underline{P}^\varepsilon & t \geq \tau(e) \text{ and } X_{\tau(e)} = x_0 - e \\ V_H & \text{otherwise} \end{cases}$$

Let $U^\varepsilon = E^H[e^{-r\tau(\varepsilon)}(V_H - P_{\tau(\varepsilon)})]$. We claim that there exists a $\underline{P}^\epsilon$ such that $(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ satisfies (8) and (9). Satisfying (9) is trivial: any $\underline{P}^\varepsilon \leq V_H$ suffices.[24] For (8), constructing the payoffs $F_B^H(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ and $F_L(\mathcal{C}_\varepsilon, \tau(\varepsilon))$, it is straightfoward to verify that both F_L and $-F_B^H$ are linearly increasing in $\underline{P}^\varepsilon$ with respective slopes $\frac{e^{(q_1+q_2)\varepsilon}}{e^{q_1\varepsilon}+e^{q_2\varepsilon}} > \frac{e^{(u_1+u_2)\varepsilon}}{e^{u_1\varepsilon}+e^{u_2\varepsilon}}$. Hence, there exists a unique P_0^ε satisfying (8). To see that this $P_0^\varepsilon \leq V_H$ (and thus is consistent with (9)), observe that at $\underline{P}^\varepsilon = V_H$: F_L is strictly positive whereas $-F_B^H = 0$, which therefore implies the intersection (i.e., P_0^ε) must lie below V_H. $\square$

A.3.1 Proofs for Signaling-game Model (Section 4.2)

Taking as given the acquirer's solution to the due diligence subgame, and resulting value functions F_B, F_H, F_L, we consider the first-stage signaling game. Let the seller's utility from posting a contract $C = (U, P)$, which results in a belief $\tilde{q}(C)$ be $u_\theta(C, \tilde{q}(C))$. If the contract is accepted, $u_\theta(C, \tilde{q}(C)) = U + F_\theta(\tilde{q}(C)|P)$; if it is rejected $u_\theta(C, \tilde{q}(C)) = 0$. A posted contract will be accepted if and only if $U \leq F_B(\tilde{q}(C)|P)$. Let $B_\theta(C, u) = \{q : u_\theta(C, q) \geq u\}$ and $\mathbb{S}_\theta$ be the support of the type-θ seller's strategy.

We use σ to denote an arbitrary strategy profile accompanied by the on-path beliefs uniquely determined via Bayes rule, and in a slight abuse of notation, $u_\theta(\sigma)$ to be the payoff of the type-θ seller in σ. We say that σ is an equilibrium if there exist off-path beliefs that support σ by (i) creating no incentives for any player to deviate and (ii) satisfying the requirements for PBE (Fudenberg and Tirole, 1991, pp. 331-333).

Lemma A.4. *In any PBE σ: (i) if $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, then σ is trivial; and (ii) $u_L(\sigma) = 0$ if and only if σ is trivial.*

Proof. For (i), fix a PBE σ such that there exists $C \in \mathbb{S}_L$, but $C \notin \mathbb{S}_H$. Then $\tilde{q}(C) = 0$. Hence, on the path, if C is posted, the deal will never be executed. This is because there are negative gains from trade when $\theta = L$: if there were positive probability of trade, then

[24] For any $\underline{P}^\varepsilon < V_H$, it will be strictly optimal to execute at $\tau(\varepsilon)$ if $q_{\tau(\varepsilon)} = q_0 - \varepsilon$ since the price is fixed thereafter and delay is costly. The acquirer strictly prefers not to execute before $\tau(\varepsilon)$, since the payoff from doing so is zero and there is positive probability of a positive payoff (i.e., if $q_{\tau(\varepsilon)} = q_0 - \varepsilon$). And the acquirer is willing to execute at $\tau(e)$ if $q_{\tau(\varepsilon)} = q_0 + \varepsilon$ since her payoff from any stopping rule thereafter is zero.

$F_L(0|P) + F_B(0|P) < 0$, and (since U is just a transfer) at least one player earns a negative payoff and would do better to deviate. So, $u_L(\sigma) = 0$, and σ is trivial by (ii).

For (ii), it is immediate that if σ is trivial, then $u_L(\sigma) = 0$. Now suppose that σ is nontrivial. Then there is positive probability of deal execution when $\theta = H$. So there is some $C' = (U', P') \in \mathbb{S}_H$ that is accepted. Hence,

$$u_H(\sigma) = u_H(C', \tilde{q}(C')) = U' + \mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_H) \geq 0,$$

with $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$. Moreover, $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] \geq \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$, where the first inequality follows from $\tau(P')$ being a threshold policy and the second inequality from $\mathcal{Q}^H$ and $\mathcal{Q}^L$ being equivalent measures. Therefore, with $U' \geq 0$ and $k_L < k_H$, we have

$$u_L(\sigma) \geq u_L(C', \tilde{q}(C')) = U' + \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_L) > 0.$$

□

Proof of Proposition 7. By definition, in any separating PBE, $\mathbb{S}_L \cap \mathbb{S}_H = \emptyset$. Hence, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the result follows from Lemma A.4. □

The High-type Optimal (HTO) Contract

To begin, we characterize the high-type optimal equilibrium restricting to pure-strategy, full pooling PBE. We then prove Proposition 8 using a series of lemmas. Propositions 9-11 follow.

In any pure-strategy, full pooling PBE $\mathbb{S}_L = \mathbb{S}_H = \{C\}$ for some contract C, and $\tilde{q}(C) = q_0$. As discussed in the text, high-type optimality requires that in the contract $C = (U, P)$ we have $U = F_B(q_0|P)$ and P solves (4). We refer to such solutions as *high-type optimal (pooling) contracts*, and use P_H to denote the price component of the solution. As with the proofs for the baseline model, analysis is aided by working with the log-likelihood of belief.

First, we show that if $z < b(k_H)$ then $P_H(z) > k_H$ and there is inefficient delay in equilibrium, compared to the social optimum, as in the baseline model (Proposition A.1). This result directly implies that there exists a $\underline{z}^*_H \geq b(k_H)$ such that for $z < \underline{z}^*_H$ (too much) delay takes place in equilibrium. Second, we show that there exists a $\bar{z}^*_H$ such that for $z \geq \bar{z}^*_H$ direct completion of the deal is optimal (Lemma A.5). Finally, we prove existence of a lower bound on the high-type seller's reservation value k_H, which if satisfied implies $\bar{z}^*_H = \underline{z}^*_H = b(k_H)$ (Lemma A.6).

Proposition A.1. *In a high-type optimal contract, if $q_0 < b(k_H)$ then $P_H(q_0) > k_H$, and hence, execution is delayed.*

Proof. First, define the high-type's payoff for $z < b(P)$ as

$$\begin{aligned} F(z|P) &\equiv F_H(z|P) + F_B(z|P) \\ &= (P - k_H)e^{(u-1)z}\left(\frac{Pu}{(u-1)(V_H - P)}\right)^{1-u} + \frac{Pe^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}}{(u-1)\left(e^z + 1\right)} \end{aligned}$$

with $u > 1$ and $F_P(z|P) \equiv \frac{\partial F(z|P)}{\partial P}$. The proof is handled in three steps corresponding to three different regions of the parameter space. Figure 10 illustrates the region of the parameter space for which each different step proves that $F_P(z|P) > 0$.

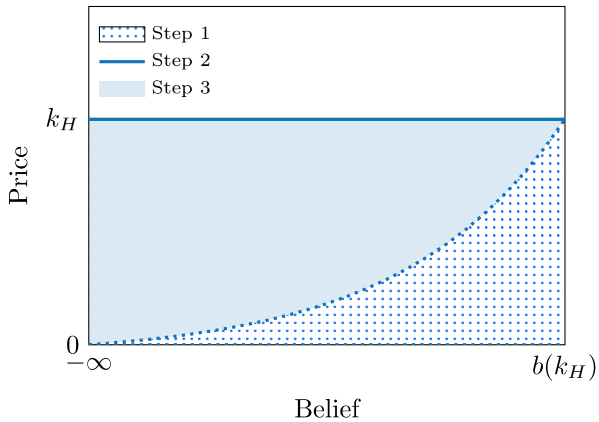

Figure 10: Illustration of where in the State Space the Different Steps prove that $F_P(z|P) > 0$.

Step 1. *For $z < b(k_H)$, we have that $F_P(z|P)|_{P=P_0(z)} > 0$.*

We can verify it for $P \in (0, k_H)$ using $b(P)$. Because $u > 1$, direct calculation yields

$$F_P(z|P)|_{z=b(P)} = \frac{(u-1)V_H(P-k_H)}{P(P-V_H)} > 0.$$

Step 2. *For $z < b(k_H)$, we have that $F_P(z|P)|_{P=k_H} > 0$.*

The proof of the previous step also implies that $F_P(z|k_H)|_{z=b(k_H)} = 0$. For $z < b(k_H)$, we have that

$$F_P(z|k_H) = -\frac{e^{(u-1)z}\left(\frac{k_H u}{(u-1)(V_H-k_H)}\right)^{-u}((u-1)e^z(k_H-V_H)+k_H u)}{(u-1)\left(e^z+1\right)\left(k_H-V_H\right)},$$

which has the same sign as $(u-1)e^z(k_H - V_H) + k_H u$. This last equation is decreasing in z. Therefore, this equation is positive for $z < b(k_H)$ and we have that $F_P(z|k_H) > 0$, for $z < b(k_H)$.

Step 3. *For $z < b(k_H)$, we have that for $P \in (P_0(z), k_H)$*

$$F_P(z|P)$$

switches sign at most once when increasing P from $P_0(z)$ until k_H for a given z.

Proving this step has several sub-steps:

1. For $z < b(P)$, we have that $F_P(z|P)$ is

$$-\frac{e^{(u-1)z}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}\left((u-1)e^z\left(V_H(V_H-k_Hu)+P^2+P(u-2)V_H\right)+u\left(V_H(k_H(-u)+k_H+P(u-2))+P^2\right)\right)}{(u-1)(e^z+1)(P-V_H)^2},$$

   which for $P < V_H$ has the same sign as

$$M(z,P)=-\left((u-1)e^z\left(V_H(V_H-k_Hu)+P^2+P(u-2)V_H\right)+u\left(V_H(k_H(-u)+k_H+P(u-2))+P^2\right)\right).$$

   This is a quadratic equation in P and therefore has at most two prices at which it is zero. Furthermore, when $P \to \pm\infty$ this equation becomes negative since the terms that contain P^2 have negative constants in front of them.

2. At $P = k_H$, this equation becomes

$$M(z, k_H) = (k_H - V_H)\left(-\left((u-1)e^z(k_H - V_H) + k_H u\right)\right),$$

   which is decreasing in z. Furthermore, $M(b(k_H), k_H)$ is 0. This result implies that for $z < b(k_H)$ $M(z, k_H) > 0$ is positive. Therefore, for $z < b(k_H)$ and $P \in (0, k_H)$ the function $M(z, k_H)$ crosses zero at most once when changing P. This result directly implies that for $z < b(k_H)$ and $P \in (P_0(z), k_H)$ the function $F_P(z|P)$ crosses zero at most once when changing P.

From Step 1 and Step 2, we know that for any $z < b(k_H)$ at $P = P_0(z)$ and at $P = k_H$ $F_P(z|P) > 0$. This result in combination with the result from Step 3 implies that $F_P(z|P) > 0$ for $z < b(k_H)$ and $P \in (P_0(z), k_H)$, which proves Proposition A.1.

Remark: For $z < b(k_H)$ and $P \in (P_0(z), k_H)$ $F_P(z, P) = 0$ cannot occur because this would imply $M(z, P)$ only touches zero and does not cross it, which is impossible since it is a quadratic equation with negative limits and $M(z, k_H) > 0$. □

Lemma A.5. *There exists a $\bar{z}_H^*$ such that for $z \geq \bar{z}_H^*$ direct completion is the high-type optimal (pooling) contract.*

Proof. We know that

$$F(z|P) \leq F_S(z|P) + \mathbb{E}_z^H\left[e^{-r\tau(P)}(V_\theta - P)\right] \leq \sup_\tau \mathbb{E}_z^H\left[e^{-r\tau}(V_\theta - k_H)\right]$$

because the dynamics of beliefs are more positive under the high-type seller's beliefs than under the buyer's beliefs and a threshold completion strategy is used by the buyer. Under high-type seller's beliefs

$$dZ_t = \frac{\phi^2}{2}dt + \phi dB_t^H.$$

The operator for this optimal stopping problem is

$$\mathcal{A}^H(z,t) = e^{-rt}\left(-r\left(\frac{e^z}{1+e^z}V_H - k_H\right) + \frac{\phi^2}{2}\left(\frac{e^z}{(1+e^z)^2}V_H - \frac{e^z(e^z-1)}{(1+e^z)^3}\right)V_H\right).$$

Observe that $\mathcal{A}_z^H(z,t)$ has the same sign as

$$\mathcal{A}_z^H(z,t)\frac{e^{rt}(1+e^z)^4}{e^z} = V_H\left((1-2e^z)\,\phi^2 - r\,(e^z+1)^2\right).$$

This function is decreasing in z and therefore switches sign at most once. Furthermore, $\lim_{z\to-\infty}\mathcal{A}^H(z,t) = e^{-rt}rk_H > 0$ and therefore $\mathcal{A}^H(z,t)$ crosses zero at most once with respect to z. This result directly implies that a threshold stopping rule is optimal (Dixit et al., 1994, Ch. 5). Furthermore, above this threshold direct completion is the high-type optimal contract since in that case

$$F(z|P) = \sup_\tau \mathbb{E}_z^H\left[e^{-r\tau}(V_\theta - k_H)\right] = \mathbb{E}_z\left[V_\theta\right] - k_H.$$

□

Lemma A.6. *Assume* $\frac{k_H}{V_H} \geq \left(1-\frac{u}{2}\right)$*. Then* $\bar{z}_H^* = \underline{z}_H^* = b(k_H)$*.*

Proof. We know that for $z < b(P)$ and $P < V_H$ $F_P(z|P)$ has the same sign as $M(z,P)$, which is defined in Proposition A.1. At $P = k_H$, $M(z,P)$ becomes

$$M(z,k_H) = (k_H - V_H)\left(-\left((u-1)e^z(k_H-V_H)+k_Hu\right)\right),$$

which is decreasing z. Furthermore, $M(b(k_H),k_H)$ is 0. Therefore, for $z > b(k_H)$ $M(z,k_H) < 0$ is negative. As a result for $z > b(k_H)$ and $P > k_H$ the function $M(z,k_H)$ crosses zero either never or twice if we change P for a given z since its limits with respect to P are negative.

Observe that

$$\frac{\partial M(z,P)}{\partial P} = -\left((u-1)e^z+u\right)(2P+(u-2)V_H),$$

which is negative for $P > k_H$ because $2P+(u-2)V_H > 2k_H+(u-2)V_H > 0$. Therefore, for $z > b(k_H)$ and $P > P_0(z) > k_H$ we have that $M(z,P) < 0$ and therefore that $F_P(z,P) < 0$. This result implies that delay is never optimal for $z > b(k_H)$.

Proposition A.1 in combination with the result from the previous step finishes the proof.

□

Lemma A.7. *If there is delay at* z_1 *and* $z_2 > z_1$ *then*

$$P_H(z_1) > P_H(z_2).$$

Furthermore, P_H *is continuous in* z *if there is delay in some neighborhood around* z*.*

Proof. If there is delay then we must have that for some $P \in (P_0(z), V_H)$

$$M(z,P) > 0.$$

Furthermore, at V_H

$$M(z,P) = (1+e^z)(u-1)u(k_H-V_H)V_H < 0$$

and $M(z,P)$ becomes negative when $P \to \pm\infty$.

The previous step implies that if there is delay then $P_H(z)$ is the largest solution of $M(z,P)=0$, which has the same sign as the derivative with respect to P of the high-quality seller's payoff. This result implies that the discriminant of $M(z,P)$, a quadratic equation with respect to P, is positive

$$uV_H\left((u-1)e^z+u\right)\left(4k_H(u-1)\left(e^z+1\right)+(u-4)V_H\left((u-1)e^z+u\right)+4V_H\right)>0,$$

and therefore

$$\left(4k_H(u-1)\left(e^z+1\right)+(u-4)V_H\left((u-1)e^z+u\right)+4V_H\right)>0,$$

which we will use in the next step.

The largest solution to $M(z,P)=0$ is

$$\frac{\sqrt{u}\sqrt{V_H}\sqrt{4k_H(u-1)\left(e^z+1\right)+(u-4)V_H\left((u-1)e^z+u\right)+4V_H}}{2\sqrt{(u-1)e^z+u}}-\frac{uV_H}{2}+V_H.$$

and the derivative of this solution with respect to z is negative

$$\frac{(u-1)\sqrt{u}\sqrt{V_H}e^z(k_H-V_H)}{\left((u-1)e^z+u\right)^{3/2}\sqrt{4k_H(u-1)\left(e^z+1\right)+(u-4)V_H\left((u-1)e^z+u\right)+4V_H}}<0,$$

which shows our result.

Continuity follows from the fact that the first-order optimality condition is continuous in z. □

The High-type Optimal (HTO) Equilibrium

Denote the set of high-type optimal (pooling) contracts as $\mathbb{C}_H(q_0)$ and the high type's PBE payoff from pooling on a solution as $u_H^*(q_0)$. If a contract will result in immediate execution, then the distinction between U and P is irrelevant as only $U+P$ matters. In this case, we simplify exposition by setting $P=0$, and U takes on the entire transfer. If there exist contracts that induce immediate execution given q_0, define $C_i(q_0) \equiv (\mathbb{E}_{q_0}[V_\theta],0)$ as the high-type optimal contract that results in immediate execution. If it exists, define $C_d(q_0) \equiv \arg\max_{\{C:P>P_0(q_0)\}} U + F_H(q_0|P)$, which is the high-type optimal contract that results in delay given q_0.

Lemma A.8. *The set of high-type optimal contracts has the following properties:*

1. *If $C_d(q_0)$ exists, then it is unique.*
2. *$\mathbb{C}_H(q_0)$ is one of the following:*
   (a) *$\mathbb{C}_H(q_0)=\{C_i(q_0)\}$, i.e., direct execution is uniquely optimal.*
   (b) *$\mathbb{C}_H(q_0)=\{C_d(q_0)\}$, i.e, delay is uniquely optimal*
   (c) *$\mathbb{C}_H(q_0)=\{C_i(q_0),C_d(q_0)\}$.*

3. *If* $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$*, then* $u_L(C_i(q_0), q_0)) > u_L(C_d(q_0), q_0))$*.*

4. *For any* $C \in \mathbb{C}_H(q_0)$*,* $u_\theta(C, q) > 0$ *is nondecreasing in* q*.*

5. *The problem's value* $u_H^*(q_0)$ *is increasing in* q_0*.*

Proof. Taking each in turn:

1. From the proof of Proposition A.1 it follows that the first-order derivative of the sellers payoff (assuming there is delay) has the same sign as a function that is a second-order polynomial in P. This implies that for $P > P_0(q_0)$ there can be at most one local maximum for $P \in (P_0(q_0), V_H)$. The fact that there is one local maximum for $P > P_0(q_0)$ proves the result

2. This result follows directly from the previous result.

3. We know that

$$\begin{aligned} u_L(C_i(q_0), q_0) + (k_L - k_H) &= u_H(C_i(q_0), q_0) \\ &= u_H(C_d(q_0), q_0) \\ &> u_L(C_d(q_0), q_0) + \mathbb{E}_{q_0}^L\left[e^{-r\tau(P)}(k_L - k_H)\right]. \end{aligned}$$

The first equality follows from the fact that payoffs only differ because of the different reservation values. The second equality follows from the fact that the high-type seller is indifferent, and the third equality follows from the fact that $P \geq k_H > k_L$ and therefore worse dynamics of beliefs make the seller worse off (the upfront is fixed). This result implies that

$$u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0) + \mathbb{E}_{q_0}^L\left[e^{-r\tau(P)}(k_L - k_H)\right] - (k_L - k_H) > u_L(C_d(q_0), q_0).$$

The last inequality follows from the fact that $\tau(P) > 0$ and $k_H > k_L$ and therefore

$$\mathbb{E}_{q_0}^L\left[e^{-r\tau(P)}(k_L - k_H)\right] > (k_L - k_H).$$

4. Because $C \in \mathbb{C}_H(q_0)$, we know that $P \geq k_H > k_L$ and therefore the sellers payoff upon completion is non-negative. The upfront is fixed. Therefore, higher beliefs q weakly improve $u_\theta(C, q)$.

The fact that $P \geq k_H > k_L$ implies that the sellers payoff (without the upfront) is non-negative. Furthermore, the upfront is always positive since $P < V_H$ and therefore $q_\theta(C, q) > 0$.

5. Take any $C \in \mathbb{C}_H(q_0)$. Fixing the price $P < V_H$, the upfront is increasing in beliefs. Furthermore, the sellers expected payoff upon execution is non-decreasing in beliefs since $P \geq k_H > k_L$ therefore the seller must be better of with higher beliefs.

□

Definition 1. *Let $\mathbb{H}$ be the set of pure-strategy, full pooling PBE in which both types pool on some contract $C \in \mathbb{C}_H$.*

Lemma A.9. *If $\sigma \in \mathbb{H}(q_0)$, then σ is a nontrivial PBE that satisfies divinity.*

Proof. Fix a $\sigma \in \mathbb{H}(q_0)$ with $C^* \in \mathbb{C}_H$ as the pooling contract. Specify the off-path beliefs as follows. For any $C' \neq C^*$: $\tilde{q}(C') = q_0$ if $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, and $\tilde{q}(C') = 0$ otherwise, where $\subset$ denotes strict inclusion. To verify σ is a PBE, note that neither type wishes to deviate to C' with $\tilde{q}(C') = 0$. In addition, the high type has no incentive to deviate to C' with $\tilde{q}(C') = q_0$ as $u_H(C', \tilde{q}(C')) = u_H(C', q_0) \leq u_H^*(q_0) = u_H(\sigma)$. By construction, at such C', $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, so the low type has no incentive to deviate either. Moreover, since $u_H(\sigma) = u_H^*(q_0) > 0$ (by Lemma A.8(4)) the equilibrium is nontrivial, and the beliefs are constructed to satisfy divinity. □

Lemma A.10. *$\mathbb{H}(q_0)$ is the set of HTO equilibria.*

Proof. By definition, $u_H(\sigma^*) = u_H^*(q_0) > 0$ for any $\sigma^* \in \mathbb{H}(q_0)$. Consider now PBE $\sigma \notin \mathbb{H}(q_0)$. Let $\underline{q}_\theta = \min_{\mathbb{S}_\theta} \tilde{q}(C)$. If $\underline{q}_H < q_0$, then there exists $C \in \mathbb{S}_H$ such that,

$$u_H(\sigma) = u_H(C, \underline{q}_H) \leq u_H^*(\underline{q}_H) < u_H^*(q_0),$$

where the last inequality is from Lemma A.8(5). If $\underline{q}_H > q_0$, then Bayesian consistency requires $\underline{q}_L = 0$. Therefore, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the equilibrium is trivial by Lemma A.4, so $u_H(\sigma) = 0 < u_H^*(q_0)$. This establishes that $u_H(\sigma) < u_H^*(q_0)$ for any PBE σ in which $\underline{q}_H \neq q_0$.

So, if $u_H(\sigma) \geq u_H^*(q_0)$, then $\underline{q}_H = q_0$. Bayesian consistency then requires $\tilde{q}(C) = q_0$ for all $C \in \mathbb{S}_H \cup \mathbb{S}_L$, meaning both types play the same strategy. By definition, to achieve $u_H^*(q_0)$ their common support must be a subset of $\mathbb{C}_H(q_0)$. If $\mathbb{C}_H(q_0)$ is a singleton, this concludes the proof. Otherwise, by Lemma A.8(2), $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$. In this case, the seller cannot mix between the two solutions, because $u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0)$ by Lemma A.8(3). □

Lemma A.11. *If $\sigma \in \mathbb{H}(q_0)$, then σ is undefeated.*

Proof. Let $\sigma \in \mathbb{H}(q_0)$ with off-path beliefs as specified in the proof of Lemma A.9. For the purpose of contradiction, suppose there exists PBE σ' that defeats σ. By Lemma A.10, $u_H(\sigma) \geq u_H(\sigma')$. The following are then required for σ' to defeat σ: there exist $C' \in \mathbb{S}'_L$ where $C' \neq C^*$, and $u_L(\sigma') > u_L(\sigma) \geq 0$. Hence, by Lemma A.4, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C' \in \mathbb{S}'_H$ as well. For σ' to defeat σ then further requires that $u_H(\sigma') \geq u_H(\sigma)$. So, we have $u_H(\sigma') = u_H(\sigma)$. In this case, σ' defeating σ requires that $\tilde{q}(C') \notin [0, q_0]$. However, as specified above, $\tilde{q}(C) \in [0, q_0]$ for all C, which is a contradiction. □

Lemma A.12. *If there exist PBEs $\sigma \in \mathbb{H}$ and $\sigma' \notin \mathbb{H}$, such that $u_L(\sigma') > u_L(\sigma)$, then σ' does not satisfy Divinity.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* \in \mathbb{C}_H$. From Lemma A.8(4), $u_\theta(C^*, q)$ is nondecreasing in q for both θ. Hence, for both types, $B_\theta(C^*, u_\theta(\sigma'))$ is an interval $[\underline{b}_\theta, 1]$, where $\underline{b}_\theta$ denotes the lowest q-value such that $u_H(C^*, \underline{b}_\theta) \geq u_\theta(\sigma')$. In addition, $\underline{b}_H \leq q_0 < \underline{b}_L$, where the first inequality is from the hypothesis that $\sigma' \notin \mathbb{H}$ and the second inequality is from the hypothesis that $u_L(\sigma') > u_L(\sigma)$. If C^* is off-path under σ', Divinity then requires that $\tilde{q}(C^*) \geq q_0$ and the high type would profit by deviating to C^*, breaking the PBE. Hence, it is sufficient to establish that C^* is off-path under σ'.

Suppose C^* is on-path in σ'. By $u_L(\sigma') > u_L(\sigma) > 0$ and Lemma A.4, σ' is nontrivial, so $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and $u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$. From Lemma A.8(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma A.4), which is a contradiction. □

Lemma A.13. *If there exist nontrivial PBEs $\sigma \in \mathbb{H}$ and $\sigma' \notin \mathbb{H}$, such that $u_L(\sigma') \leq u_L(\sigma)$, then σ that defeats σ'.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* = (U^*, P^*) \in \mathbb{C}_H(q_0)$. By definition, σ defeats σ' if: (i) C^* is off-path in σ'; (ii) $u_\theta(\sigma) \geq u_\theta(\sigma')$ for both θ, and holding strictly for at least one θ; and (iii) $\tilde{q}'(C^*) \notin [q_0, 1]$. Requirement (ii) holds for $\theta = H$ by Lemma A.10 and for $\theta = L$ by hypothesis. Moreover, if requirement (iii) fails, then because $u_H(C^*, q)$ is nondecreasing in q (Lemma A.8(4)),

$$u_H(\sigma') \geq u_H(C^*, \tilde{q}'(C^*)) \geq u_H(C^*, q_0) = u_H(\sigma),$$

which contradicts Lemma A.10. Hence, it is sufficient to show that (i): C^* is off-path in σ'.

Suppose C^* is on-path in σ'. Since σ' is nontrivial, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and $u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$, where the inequality is from Lemma A.10. From Lemma A.8(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma A.4), which is a contradiction. □

Proof of Proposition 8. Follows from Lemmas A.9-A.13 □

Proof of Proposition 9. Follows from Proposition 8, Proposition A.1, Lemma A.5, and Lemma A.6. □

Proof of Proposition 10. Follows from Proposition 8, Proposition A.1, Lemma A.5, Lemma A.6, and Lemma A.7. For $z = b(k_H)$ we have that $F_P(b(k_H)|k_H) = 0$ while for $P > k_H$ we have that $F_P(b(k_H)|P) < 0$ (see the proof of Lemma A.6). Furthermore, $F_P(z|P) = 0$ determines the optimal price for $z < b(k_H)$ and this function is continuous in z and P, which proves the limiting result for the price. The result for the upfront follows from the fact that $U^*_H(z) = F_B(z|P^*_H(z))$ is continuous in z and P and increasing in z and decreasing in P. □

Proof of Proposition 11. Follows from Proposition 8, Proposition A.1, and Lemma A.5. □

A.4 Extensions

Proof of Proposition 13. Assume the acquirer executes the deal for a belief q then in equilibrium the price must be $P_0(q)$. Assume the price is $P \neq P_0(q)$. If $P > P_0(q)$ the acquirer optimally delays execution. If $P < P_0(q)$ then competitive buyers would drive up the price to $P_0(q)$ at the time the seller accepts an offer. Therefore, if a deal is executed the price must be $P_0(q)$.

If for some belief q the seller wants to execute the deal but no buyer has made an appealing offer then each buyer has an incentive to deviate and offer the seller $P_0(q)$. Assume this is not the case. For $q \geq q^*$ the seller prefers to receive an offer $P_0(q)$. Let $\mathcal{Q} \subset [q^*, 1)$ be the set of beliefs for which buyers make offers $P_0(q)$ and $\mathcal{Q}'$ the set of beliefs for which they do not. Take any $q \in \mathcal{Q}'$ then we know there exists a $\hat{q} > q$ such that $\hat{q} \in \mathcal{Q}$. If this was not the case then when beliefs converge to one both buyers' and seller's value would converge to zero since the chance of the deal happening within finite time goes to zero. Therefore, at some point buyers would have an incentive to offer a price $P \in (k, P_0(q))$ but if this price is being offered then competition drives it up to $P_0(q)$.

It must be the case that $\inf_q \mathcal{Q} = q^*$ else a buyer has an incentive to make an offer at q^* and have the ability to buy the asset at a price $P_0(q^*) < P_0(\inf_q \mathcal{Q})$ (remember the buyer can always perform due diligence until beliefs reach $\inf_q \mathcal{Q}$).

The only remaining option is that there exists some interval $(q_1, q_2) \subseteq \mathcal{Q}'$ with $q_1, q_2 \in \mathcal{Q}$ and $q^* < q_1 < q_2 < 1$.[25] Assume this is the case then we know that at q_2 the buyer's payoff is less than $V_H < \infty$. Furthermore, as $q \downarrow q_1$ we know that each buyer's payoff converges to $F_B(q_1|P_0(q_1))/2$ (there are two of them). Therefore, there exists a $\hat{q} \in (q_1, q_2)$ close to q_1 such that each buyer prefers to offer $P_0(\hat{q})$ and be the winning buyer for sure since

$$0 < \frac{F_B(q_1|P_0(q_1))}{2} < \lim_{q \downarrow q_1} F_B(q|P_0(q)).$$

Therefore, this interval $(q_1, q_2) \subseteq \mathcal{Q}'$ cannot exist and we must have that $\mathcal{Q} = [q^*, 1]$ and in equilibrium the seller can execute the deal whenever he wants at $P_0(q)$.

The seller's payoff therefore boils down to

$$\sup_{\tau} \mathbb{E}_z \left[e^{-r\tau} \left(P_0\left(q_\tau\right) - k \right) \right] = F_S(q|P_S(q)),$$

which shows the result. □

Lemma A.14. *Suppose that $k < V_L < V_H$. Then, there exists a $\underline{\gamma} > 0$ such that if and only if $\phi^2/r = \gamma \leq \underline{\gamma}$, then for any z_0 the winning offer is $P_0(z_0)$ and the transaction is executed immediately.*

Proof. Observe that the seller's payoff for $z \leq b(P)$ can be written as

$$F_S(z|P) = \psi(P, u) \frac{e^{uz}}{1 + e^z}$$

[25] Single points at which buyers do not offer $P_0(q)$ do not alter the seller's payoff or execution time since beliefs and therefore prices move continuously.

with

$$\psi(P,u) = (P-k)\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^{-u}\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}+1\right).$$

Given any z, picking the optimal price boils down to solving

$$\max_P F_S(z|P) \iff \max_{P\geq P_0(z)} \psi(P,u) \tag{11}$$

since the seller either picks the highest price that leads to direct execution $P_0(z)$ or a higher price that leads to delay.

For any belief it is optimal to have direct execution (pick $P_0(z)$) if and only if

$$\psi_P(P,u) \leq 0.$$

Assume that $\psi_P(P,u) \leq 0$ then the optimal solution to equation (11) is $P_0(z)$. Assume that direct execution is optimal for all z but $\psi_P(P,u) > 0$ for some $\hat{P}$. Take $\hat{z}$ such that $\hat{P} = P_0(\hat{z})$ then for a small $\epsilon > 0$

$$\psi(\hat{P}+\epsilon, u) > \psi(\hat{P}, u)$$

since $\psi_P(P,u)$ is continuous in P and $\psi_P(\hat{P},u) > 0$. This result contradicts the fact that direct execution is optimal for $\hat{z}$. Therefore, if direct execution is optimal $\psi_P(P,u) \leq 0$.

Observe that $\psi_P(P,u)$ for $P \in (V_L, V_H)$ has the same sign as

$$\begin{aligned}
\tilde{\psi}_P(P,u) =& \psi_P(P,u)(u-1)(P-V_H)^2(P-V_L)\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^u \\
=& k(u-1)u(V_H-V_L)^2 - P^3 + P^2(-uV_H + uV_L + 2V_H + V_L) \\
& - P\left(2\left(-u^2+u+1\right)V_HV_L + u^2V_L^2 + (u-1)^2V_H^2\right) + V_HV_L(-uV_H + uV_L + V_H).
\end{aligned}$$

Observe that for $P \in (V_L, V_H)$

$$\begin{aligned}
\tilde{\psi}_{P,u,P}(V_L,u) &= -2(u-1)(V_H-V_L)^2 < 0, \\
\tilde{\psi}_{P,u,P,P}(P,u) &= -2(V_H-V_L) < 0, \\
\tilde{\psi}_{P,u,P}(P,u) &= -2(V_H-V_L)(P+(u-1)V_H-uV_L) < 0, \\
\tilde{\psi}_{P,u}(V_L,u) &= (2u-1)(k-V_L)(V_H-V_L)^2 < 0.
\end{aligned}$$

This result implies that $\tilde{\psi}_{P,u}(P,u) < 0$ for $P \in (V_L, V_H)$.

As a result of the previous step, if for some $\hat{u}$ $\tilde{\psi}_P(P,\hat{u}) \leq 0$ for $P \in (V_L, V_H)$ then for any $u \geq \hat{u}$ $\tilde{\psi}_P(P,u) \leq 0$ for $P \in (V_L, V_H)$.

The relationship between $\psi_P(P,u)$ and $\tilde{\psi}_P(P,u)$ then implies that if for some $\hat{u}$ $\psi_P(P,\hat{u}) \leq 0$ for $P \in (V_L, V_H)$ then for any $u \geq \hat{u}$ $\psi_P(P,u) \leq 0$ for $P \in (V_L, V_H)$.

It is easy to see that

$$u = \frac{1+\sqrt{8\frac{r}{\phi^2}+1}}{2} = \frac{1+\sqrt{\frac{8}{\gamma}+1}}{2}$$

is decreasing in γ. Therefore, if for some $\hat{\gamma}$ $\psi_P(P, u(\hat{\gamma})) \le 0$ for $P \in (V_L, V_H)$ then for any $\gamma \le \hat{\gamma}$ $\psi_P(P, u(\gamma)) \le 0$ for $P \in (V_L, V_H)$. The previous steps show that direct execution is optimal for any belief if and only if $\gamma \le \underline{\gamma}$.

We can collect the terms related to u in $\tilde{\psi}_P(P, u)$ and obtain

$$\begin{aligned}\tilde{\psi}_P(P,u) = &-u(V_H - V_L)\left(k(V_H - V_L) + P^2 - 2PV_H + V_H V_L\right)\\ &+ u^2(k-P)(V_H - V_L)^2 - P^3 + 2P^2 V_H + P^2 V_L - PV_H^2 - 2PV_H V_L + V_H^2 V_L.\end{aligned}$$

As $\gamma \to 0$, $u \to \infty$ and

$$\tilde{\psi}_P(P,u) \to u^2(k-P)(V_H - V_L)^2 < 0.$$

Therefore, there exists a γ small enough such that $\tilde{\psi}_P(P, u) < 0$ and direct execution is optimal. □

Lemma A.15. *Suppose that $k < V_L < V_H$ and $\phi^2/r > \underline{\gamma}$ then there exist two thresholds $z_a < z_b$ such that*

(a) For $z_0 \in (z_a, z_b)$, the winning offer is $P_0(z_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(z_b))$.

(b) For $z_0 \notin (z_a, z_b)$, the winning offer is $P_0(z_0)$ and the transaction is executed immediately.

Proof. Assume $\gamma > \underline{\gamma}$ then we know delay takes places for some z, see Lemma A.14. Assume delay takes place for $\hat{z}$ and the seller picks a price $\tilde{P} > P_0(\hat{z})$ then define $\tilde{z}$ as the solution to $\tilde{P} = P_0(\tilde{z})$. We then know that delay must take place for $z \in (\hat{z}, \tilde{z})$ since

$$\psi(\tilde{P}, u) = \max_{P \ge P_0(\hat{z})} \psi(P,u) \ge \max_{P \ge P_0(z)} \psi(P,u).$$

Assume there is more than one region in which delay takes place. Assume for simplicity there are two. The proof also works with more regions. The existence of these two delay regions implies that there exists a $z_1 < z_2 < z_3 < z_4 < z_5$ such that

1. For $z < z_1$, direct execution is optimal because $\lim_{P\to V_L} \psi(P,u) = \infty$ and $\psi(P,u) < \infty$ for any $P \in (V_L, V_H)$ and therefore for $z < z_1$ $\psi_P(P_0(z), u) \le 0$.
2. For (z_1, z_2), delay takes place and therefore there exists a $z \in (z_1, z_2)$ such that $\psi_P(P_0(z), u) > 0$.
3. For (z_2, z_3), no delay takes place and therefore for $z \in (z_2, z_3)$ $\psi_P(P_0(z), u) \le 0$.
4. For (z_3, z_4), delay takes place and therefore there exists a $z \in (z_3, z_4)$ such that $\psi_P(P_0(z), u) > 0$.

5. For $z > z_5$, no delay takes since $\lim_{P \to V_H} \psi_P(P,u) = (u-1)u(k - V_H)(V_H - V_L)^2 < 0$

This implies that $\psi_P(P,u) = 0$ at least 5 values of $P \in (V_L, V_H)$.

We know that $\hat{\psi}_P(P,u)$ has the same sign as $\psi_P(P,u)$. Observe that $\hat{\psi}_P(P,u)$ is a third-degree polynomial in P, which has at most three zero solutions and therefore it must be the case that we only have one region in which delay takes place. □

Proof of Proposition 14. The result follows directly from Lemma A.14 and Lemma A.15. □

Proof of Proposition 15. For $\gamma > \underline{\gamma}$, delay is optimal and dynamic bidding strictly dominates static bidding for some beliefs. Therefore, there must exist a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$ dynamic bidding strictly dominates static bidding. □

Proof of Proposition 16. We first want to show that the value function $F_B(q|P)$ is increasing in q. Assume this is not the case and $F_B(q|P) > F_B(q'|P)$ while $q < q'$. Fix a sample path of the information process $\{X_t\}_{t \geq 0}$ and let $\{q_t\}_{t \geq 0}$ be the beliefs this information generates when starting from q. Apply the due diligence quality and stopping rule used for $\{q_t\}_{t \geq 0}$ to construct a beliefs process $\{\hat{q}'_t\}_{t \geq 0}$ which starts from q' instead. In this situation, $q_t < \hat{q}'_t$ and therefore

$$\mathbb{E}\left[V_\theta - P | q_{\tau(P)}\right] < \mathbb{E}\left[V_\theta - P | \hat{q}'_{\tau(P)}\right].$$

This result holds for every sample path of information and due diligence quality. The costs given $\{q_t\}_{t \geq 0}$ and $\{\hat{q}'_t\}_{t \geq 0}$ are the same. Furthermore, we are using the suboptimal due diligence quality and stopping rule when starting from q'. As a result of all of this, we must have that

$$F_B(q|P) < F_B(q'|P).$$

The previous step also implies that $F_B(q|P) > 0$ since for any $q \in (0,1)$ $F_B(q|P) \geq 0$.

In the region where the acquirer performs due diligence, her value function solves the Hamilton-Jacobi-Bellman equation

$$0 = \sup_{\phi^2 \geq 0} \left\{ -rF_B(q|P) - c(\phi^2) + \frac{\phi^2}{2} q^2 (1-q)^2 F''(q|P) \right\}.$$

For the optimal level of due diligence quality, we must have that

$$\frac{\phi^2}{2} q^2 (1-q)^2 F''(q|P) > -c(\phi^2) + \frac{\phi^2}{2} q^2 (1-q)^2 F''(q|P) = rF_B(q|P) > 0.$$

Therefore, the optimal level of due diligence quality $\phi^2 > 0$.

The first-order optimality condition implies that

$$\phi^2 = \frac{1}{2} q^2 (1-q)^2 F''(q|P)$$

and therefore that

$$
\begin{aligned}
rF_B(q|P) &= \frac{1}{8}\left(q^2(1-q)^2F''(q|P)\right)^2 = \frac{1}{2}\phi^4, \\
\phi^2 &= \sqrt{2rF_B(q|P)}.
\end{aligned}
$$

□